LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes College - - 1975 Senior Examination
(Part I)

NAME ______________________________ SCHOOL ______________________________

Directions: For each problem, write your answer in the space provided. Leave any answer which involves π or simplified radicals as it is – do not use approximations.

1. Solve:

$$3x^2 = 2x + 1$$

1. ____________________

2. An equilateral triangle has an altitude of length 3. Find the area of the triangle.

2. ____________________

3. If $\sin x = 3/4$ and $90° < x < 180°$, find $\tan x$.

3. ____________________

4. A line has slope $3/2$ and passes through the point which has coordinates $(-2,4)$. Find an equation of the line in the form $ax + by + c = 0$ where a, b, and c are integers.

4. ____________________

5. Find all values of x such that $|3 - 2x| < 7$.

5. ____________________

6. Solve simultaneously:

$$\begin{cases} 2x + y = 4 \\ x + 2y = -1 \end{cases}$$

6. ____________________

7. If $f'(x) = 2x + 3$ and $f(0) = 2$, find $f(x)$. (Note: $f'(x)$ is the derivative of $f(x)$ with respect to x.)

7. ____________________

8. If $f(x) = 2x + 1$ and $g(x) = 2x + 2$, find a rule of correspondence for $g(f(x))$.

8. ____________________

9. Let $y = 8x - x^2 - 12$. Find the largest value which y can have.

9. ____________________

10. If f is a function which has the property that $f(ab) = f(a) + f(b)$ for all positive real numbers a and b, find $f(1)$.

10. ____________________

11. Find an equation of the curve each of whose points are equidistant from the point $(4, 0)$ and the $y-axis$.

11. ____________________

12. Find:

$$\sin^{-1} \tan \frac{5\pi}{4}$$

12. ____________________

13. A tin can in the form of a right circular cylinder with lids on top and bottom has a total area of $100\pi\ sq.in.$ Express the volume, V, (in $cu.in.$), of the can as a function of r, the radius of the base. (Neglect the thickness of the tin.)

13. ______________________

14. Solve: $3\ \log x - \log 2x = 0$

14. ______________________

15. Let $f(x) = |x - 2| + |x + 2|$. Assuming that $x \geq 2$, find a and b such that $f(x) = ax + b$.

15. ______________________

16. If $0 \leq x \leq \pi/2$ and $\sin x - \sin 2x = 0$, solve for x.

16. ______________________

17. Simplify:

$$\frac{\sqrt{x2+1} - \frac{1}{\sqrt{x2+1}}}{\frac{x2}{\sqrt{x2+1}} - \sqrt{x2+1}}$$

17. ______________________

18. Write the number 25 as a number to the base 2.

18. ______________________

19. Solve: $x \cdot 2^x + 2^x$

19. ______________________

20. Find the length of a side of a rhombus whose diagonals have lengths 8 and 10.

20. ______________________

21. Solve for x if $(\sin x + \cos x)^2 = 3/2$ and $0 < x < \pi/2$.

21. ______________________

22. A man who is 6 feet tall is standing at a distance of 10 feet from the base of a lamp post which is 20 ft. high. What is the length of his shadow if cast by a light at the top of the post?

22. ______________________

Senior Examination – 1975
(Part II)

NAME ______________________ SCHOOL ______________________

1. Solve:

$$\frac{x}{x^2 - 1} = \frac{2}{x + 1}$$

1. ______________

2. An isosceles right triangle has hypotenuse with length $\sqrt{18}$. Find the volume of the solid generated by revolving the triangle about one of its legs. 2. ______________

3. Find an equation of the circle with center $(3\,, -2)$ and radius 4. 3. ______________

4. If $\cos A = 3/8$, find $\cos(-A)$. 4. ______________

5. If f is a function having a rule of correspondence $f(x) = x/2 - 3$, find a rule of correspondence for $f^{-1}(x)$, the inverse of $f(x)$. 5. ______________

6. If $\log_2 3 = a$, find $\log_3 8$ in terms of a. 6. ______________

7. Find real numbers x and y such that $(2x + 1)\,1 - 2 = y + 31$ 7. ______________

8. Given triangle ABC with a right angle at C. If $\overline{AC}$ and $\overline{BC}$ have lengths 4 and 6 respectively, and M is the midpoint of $\overline{AB}$, find the area of triangle AMC. 8. ______________

9. Find the $y - intercept$ of the line which passes through the midpoint of the line segment having endpoints $(2, -3)$ and $(6, 5)$, and is perpendicular to the line having an equation $y = 3x - 4$. 9. ______________

10. A rectangle has perimeter 200. Express the area of the rectangle as a function, $A(w)$, of the width, w, of the rectangle. 10. ______________

11. Solve:

$$x^3 - x^2 - 3x + 2 = 0$$

11. ______________

12. Find two consecutive integers, m and n such that $m < \log_3 40 < n$. 12. ______________

13. Find the area of the region between the graphs of $f(x) = x^2$ and $g(x) = |\,x|$ from $x = 0$ to $x = 1$. 13. ______________

14. If four coins are tossed, find the probability that exactly two heads will turn up. 14. ______________

15. If $f(x) = x^2 + 2x - 8$, find the set of all x for which $f(x) < 0$. 15. ______________

16. Find:

$$\sin\left(\tan^{-1}\left(\frac{-2}{3}\right)\right)$$

16. ______________________

17. Find the radius of the circle whose area is double if its radius is increased by 2 units.

17. ______________________

18. Solve:

$$\sqrt{5x-11}-\sqrt{x-3}=4$$

18. ______________________

19. A rectangular picture has an area of $144\ sq.in.$ It is surrounded by a order which is $2\ in.$ wide. If the area of the border is $120\ sq.in.$, find the dimensions of the picture.

19. ______________________

20. Find:

$$\sum_{n=1}^{100}\frac{1}{n^2+n}$$

$$\left[HINT:\ \frac{1}{n^2+n}=\frac{1}{n}-\frac{1}{n+1}\right]$$

20. ______________________

21. Find the area of the largest right triangle which can be inscribed in the circle of radius 10.

21. ______________________

22. If A and B are acute angles and $\sin A = 1/3$ and $\sin B = 2/3$, find $\sin(A+B)$.

22. ______________________

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes College - - 1975 Junior Examination
(Part I)

NAME ____ SCHOOL ____________________________________

Directions: For each problem, write your answer in the space provided. Leave any answer which involves π or simplified radicals as it is – do not use approximations.

1. Solve simultaneously:

$$\begin{cases} 2x + y = 4 \\ x + 2y = -1 \end{cases}$$

1. ______________________

2. Solve:

$$3x^2 = 2x + 1$$

2. ______________________

3. An equilateral triangle has an altitude of length 3. Find the area of the triangle.

3. ______________________

4. Find all values of x such that $|3 - 2x| < 7$.

4. ______________________

5. A line has slope $3/2$ and passes through the point which has coordinates $(-2,4)$. Find an equation of the line in the form $ax + by + c = 0$ where a, b, and c are integers.

5. ______________________

6. A circle has circumference C. Express the area, A, of the circle as a function of C.

6. ______________________

7. If $\sin x = 3/4$ and $90° < x < 180°$, find $\tan x$.

7. ______________________

8. Solve:

$$3 \log x - \log 2x = 0$$

8. ______________________

9. Solve:

$$\sqrt{2x^2 - 3x + 1} + 4 = 2x$$

9. ______________________

10. A right triangle is inscribed in a circle of radius 5. If the length of one of the legs of the triangle is 7, find the length of the other leg.

10. ______________________

11. If $f(x) = 2x + 1$ and $g(x) = 2x + 2$, find a rule of correspondence for $g(f(x))$.

11. ______________________

12. Find the circumference of a circle whose area is twice the area of a circle with circumference 6π.

12. ______________________

13. Solve:

$$\frac{2x}{x+1} + \frac{x-1}{x} = \frac{1}{x}$$

13. ______________________

14. Two boys on bicycles leave point A at the same moment, one boy heading north, the other heading east. They travel at constant speeds, one going two miles per hour faster than the other. Two hours after starting, they are 20 miles apart. How fast is the slower cyclist traveling in miles per hour?

14. ______________________

15. If f is a function with rule or correspondence

$$f(x) = \sqrt{\frac{x}{2x+1}}$$

find the domain of f.

15. ______________________

16. Find a quadratic equation with integral coefficients in the form $ax^2 + bx + c = 0$ having roots -1 and $3/5$.

16. ______________________

17. A rectangular box has square base and has lids on top and on bottom. If it has a total surface area of $1000\ sq. ft.$, express the volume V as a function of x, where x is the length of the side of the square. (Neglect the thickness of the material.)

17. ______________________

18. Simplify:

$$\frac{\sqrt{x^2+1} - \frac{1}{\sqrt{x^2+1}}}{\frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}$$

18. ______________________

19. Write the number 25 as a number to the base 2.

19. ______________________

20. Find the $y - intercept$ of the line which passes through the points having coordinates $(3, 2)$ and $(-2, 4)$.

20. ______________________

21. Find:

$$\sin^{-1} \tan 5\pi/4$$

21. ______________________

22. Let $f(x) = |x - 2| + |x + 2|$. Assuming that $x \geq 2$, find a and b such that $f(x) = ax + b$.

22. ______________________

Junior Examination – 1975
(Part II)

NAME ____ SCHOOL ____________________________________

1. If $f(x) = x2 + 2$ for all real numbers x, find $f(x + 2)$. 1. ______________________

2. A point P is $10\ in.$ from the center of a circle having radius $6\ in.$ Two distinct tangents to the circle from P have points of contact at Q and R. Find the length of $\overline{QR}$ in inches. 2. ______________________

3. If f is a function with rule or correspondence $f(x) = x/3 + 1$, find a rule of correspondence for $f^{-1}(x)$, the inverse of f(x). 3. ______________________

4. Solve simultaneously:

$$\begin{cases} x^2 - 3y^2 = -11 \\ 2x - y = 0 \end{cases}$$

4. ______________________

5. If a regular polygon has an exterior angle of measure 60°, how many sides has the polygon? 5. ______________________

6. Find:

$$\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$$

6. ______________________

7. Find an equation of the set of all points in the plane, each of which is equidistant from the two points having coordinates $(4, 1)$ and $(8, -5)$. 7. ______________________

8. If $\log_2 3 = a$, find $\log_3 8$ in terms of a. 8. ______________________

9. If $f(x) = x^2 + 2x - 8$, find the set of all x such that $f(x) < 0$. 9. ______________________

10. Solve:

$$x^3 - x^2 - 3x + 2 = 0$$

10. ______________________

11. A rectangle has perimeter 200. Express the area, A, of the rectangle as a function of the width, w, of the rectangle. 11. ______________________

12. Triangle ABC has a right angle at C. If $\tan A = 2/3$ and the length of $\overline{BC} = 5$, find the length of $\overline{AB}$. 12. ______________________

13. Find an equation of the line which is perpendicular to the $x - axis$ and passes through the point having coordinates $(3, 5)$. 13. ______________________

14. Find the length of the side of a square whose area is doubled if each of the sides is increased by 2 units. 14. ______________________

15. Find an equation of the circle which passes through the 3 points having coordinates $(0,0)$, $(0,6)$, and $(8,0)$. 15. ______________________

16. A square is inscribed in a circle of radius 10. Find the area of the square. 16. ______________________

17. If $\cos A = 3/8$, find $\cos(-A)$. 17. ______________________

18. A manufacturer produces x items at a total cost of $1000 + 20x - \frac{3x}{10}$ dollars, and sells them for $500 - x$ dollars each. Write his profit, $P(x)$, as a function of x. 18. ______________________

19. If three coins are tossed, find the probability that exactly 2 heads appear. 19. ______________________

20. If a point P having coordinates (x, y) represents any point on the graph of $y = x^2 + 1$, express the distance, d, from P to the point having coordinates $(1, -1)$ as a function of x. 20. ______________________

21. Find a solution other than $x = 3$ to the equation:

$$\log_3 x = \log_x 3$$

21. ______________________

22. Find:

$$\sin\left(2\cos^{-1}\frac{1}{3}\right)$$

22. ______________________