

LUZERNE COUNTY MATHEMATICS CONTEST
 Luzerne County Council of Teachers of Mathematics
 Wilkes College - - 1983 Junior Examination
 (Section I)

NAME _____ SCHOOL _____

Directions: For each problem write your answer in the space provided.

Do not use decimal approximations for π , $\sqrt{2}$, etc.

1. Simplify and express without negative exponents:

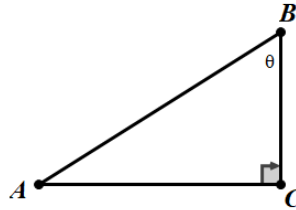
$$\frac{a^{-2}(b^2c)^{-3}}{(a^{-5}b^{-4})^2c^{-7}}$$

1. _____

2. Find all values of x which satisfy $2x^2 + 5x - 3 = 0$

2. _____

3. Given that $\cos \theta = \frac{4}{7}$ and $\overline{AB} = 2$, find the length of \overline{BC} in the right triangle ABC shown.



3. _____

4. A circle is circumscribed about a square which has sides of length 4. Find the area of the circle.

4. _____

5. Find all values of x which satisfy the equation $4^{5x} = (16)^{2x-1}$

5. _____

6. Given that $f(x) = \frac{1}{x+1}$ and $g(x) = x - \frac{1}{x}$, find $f(g(2))$.

6. _____

7. Give all values of x for which the function

$$f(x) = \frac{\log x}{x-1}$$

is defined.

7. _____

8. Give the coordinates of all points on the line $y = 2$ which are 5 units away from the point $(3, -1)$.

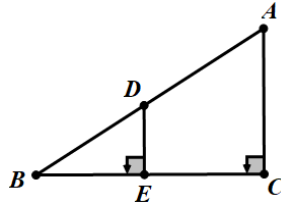
8. _____

9. Find all values of x which satisfy the inequality $3x - 7 > 5x + 7$.

9. _____

(OVER)

10. In the figure shown, $AC \perp BC$,
 $DE \perp BC$, $\overline{AC} = 15$, $\overline{DE} = 6$. If the
length of EC is p , express the
length of BE in terms of p .



11. Find all values of x which satisfy
 $\log(x + 3) + \log(x - 3) = 0$.

12. Solve the following system of equations:

$$A - B - C = -4$$

$$3A + B = 14$$

$$4A + 2B - c = 20$$

13. Find the coordinates of the midpoint of the line
segment joining P and Q where P is the point
 $(4, -7)$ and Q is the point $(8, 3)$.

14. Give the x and y intercepts of the ellipse
 $4x^2 + 9y^2 = 36$.

15. A salesman receives a base salary of \$600 per month
plus a 5% commission on all sales. Given that he
earned a total of \$1000 last month, find the amount
of his sales for the month.

16. Given that $x = 1$ is a root of the polynomial
 $2x^3 - x^2 - 7x + 6 = 0$, find the other 2 roots.

17. Suppose $f(x) = A \log_{10} x + B$ for some constants A and
B. If $f(1) = 10$ and $f(10) = 1$, find A and B.

18. Give the value of $\csc(\text{Arcsin } \frac{1}{15})$.

19. A regular 12 sided polygon is inscribed in a circle
of radius r . Find the area of the polygon in terms
of r .

20. How many even 3 digit numbers can be constructed
using the digits 3, 4, 5, 6, 7, if any of the digits
may be used more than once in any given number?

10. _____

11. _____

12. A= _____

B= _____

C= _____

13. _____

14. X int.= _____

Y int.= _____

15. _____

16. _____

17. _____

18. _____

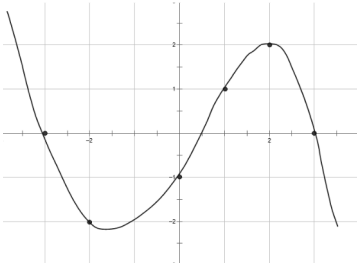
19. _____

20. _____

Junior Examination - 1983
(Section II)

NAME _____ SCHOOL _____

1. Give the radian measure of an angle with degree measure 72° . 1. _____
2. Find all pairs (x, y) which satisfy the following system of equations:
 $x^2 - y = 1$
 $2x - y = -3$ 2. _____
3. Express .0000157 as the product of an integer power of 10 and a number between 1 and 10. 3. _____
4. Given that $\log_b 2 = 1.2$ and $\log_b 3 = 1.9$, find $\log_b (\frac{8}{9})$. 4. _____
5. Consider the function f whose graph is sketched below. List the letters of all those of the following statements which are true.



- a) $f(2) \geq f(x)$ for all x
- b) $f(x) < 0$ if $-3 < x < 1$
- c) $f(-1) < f(0)$
- d) $f(-2) = -f(2)$
- e) $f(-3) = -f(3)$

5. 5. _____
6. A circle has a radius 10. Find the radius of a circle which has twice the area of the given circle. 6. _____
7. Given that $f(x) = x^2 + 1$, and $h \neq 0$, find $\frac{f(x+h) - f(x)}{h}$ and simplify. 7. _____
8. Find all values of x (in radian) in the interval $[0, 2\pi)$ which satisfy the equation $2\sin^2 x - \sin x = 1$. 8. _____
9. Find all values of x which satisfy the equation $\frac{3}{x+1} - \frac{2}{x} + 1 = 0$. 9. _____

(OVER)

10. A right triangle is inscribed in a circle having diameter 17. If one of the legs of the triangle has length 8, find the area of the triangle.

10. _____

11. A ramp is to be installed at the entrance of a classroom building. The entrance is 2 feet from the ground, and the angle between the ramp and the ground should be 10 degrees. Which of the following gives the length of the ramp?

- a) $\frac{2}{\cos 10^\circ}$ b) $\frac{\cos 10^\circ}{2}$ c) $\frac{2}{\sin 10^\circ}$ d) $\frac{\sin 10^\circ}{2}$

11. _____

12. Given that $\sin t = \frac{1}{4}$ and $\cot t < 0$, find $\cos t$.

12. _____

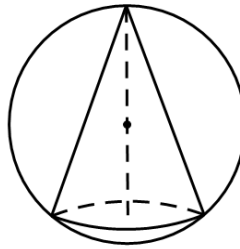
13. A regular hexagon has sides of length 2. Find its area.

13. _____

14. Determine all values of b so that the polynomial $x^2 + bx + 25$ has exactly one real root.

14. _____

15. A right circular cone is inscribed in a sphere of radius 10 as shown. Express the volume of the cone as a function r . The radius of its base.



15. _____

16. Find the coordinates of the vertex of the parabola $y = 160x - 2x^2$.

16. _____

17. If $f(0) = 1$ and $f(1) = 1$ and, for all $k \geq 2$, $f(k) = f(k-1) + f(k-2)$, find $f(4)$.

17. _____

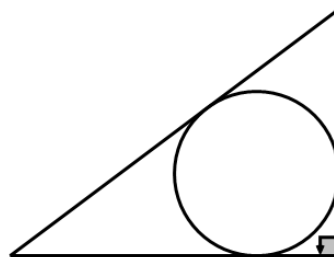
18. If the lines $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ meet at right angles, find the coordinates of their point of intersection.

18. _____

19. A set of 4 tubes in a radio consists of two good tubes and two defective ones. If 3 tubes are selected at random from this group, what is the probability that exactly two of three selected will be defective?

19. _____

20. A circle is inscribed in a right triangle whose legs have lengths 6 and 8, respectively. Find the radius of the circle.



20. _____

LUZERNE COUNTY MATHEMATICS CONTEST
 Luzerne County Council of Teachers of Mathematics
 Wilkes College - - 1983 Senior Examination
 (Section I)

NAME _____ SCHOOL _____

Directions: For each problem write your answer in the space provided.

Do not use decimal approximations for π , $\sqrt{2}$, etc.

1. Simplify and express without negative exponents:

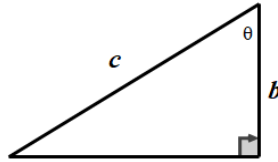
$$\frac{a^2(b^2c^3)^{-3}}{(a^{-5}b^{-4})^2c^{-7}}$$

1. _____

2. Determine A such that the point (3,-2) is on the line $Ax - 2y + 7 = 0$.

2. _____

3. Given that $\cos \theta = \frac{4}{7}$ and $c = 2$, find b in the right triangle shown.



3. _____

4. A circle is circumscribed about a square which has sides of length 4, and another circle is inscribed in the same square. Find the area of the region between the two circles.

4. _____

5. Find all values of x which satisfy the equation

$$4^{5x} = \left(\frac{1}{16}\right)^{2x-1}$$

5. _____

6. Given that $f(x) = \frac{1}{x+1}$ and $g(x) = x - \frac{1}{x}$, find $f(g(z))$.

6. _____

7. Give all values of x for which the function

$$f(x) = \frac{\log x}{x-1}$$
 is defined.

7. _____

8. Give the coordinates of all points on the line $y = 2$ which are 5 units away from the point (3,-1).

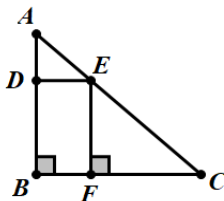
8. _____

9. Find all values of x which satisfy the inequality

$$\frac{x+1}{2x-3} > 0.$$

9. _____

10. In the figure shown, $\triangle ABC$ is a right triangles with right angle at B. DEFB is a rectangle with $\overline{DB} = 6$, $\overline{BF} = 4$. If $\overline{AD} = 3$, find \overline{AC} .



(OVER)

10. _____

11. Determine all values of x which satisfy the equation $\log_{10}x + \log_{10}(x - 3) = 1$. 11. _____
12. Given that $\frac{8x^2 - 20x + 12}{x(x-2)^2}$ can be expressed in the form $\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$, determine the constants A , B , and C . 12. A= _____
B= _____
C= _____
13. Suppose $(-4, 3)$ is the midpoint of the line segment forming P and Q , where P is the point $(8, -5)$. What are the coordinates of Q ? 13. _____
14. Find the center and radius of the circle $4x^2 + 16x + 4y^2 - 24y = 48$. 14. center= _____
radius= _____
15. The height of a tin can is 5 inches, and its lids are circles having radius 4 inches. Find the total surface area of the can (including lids) in square inches. 15. _____
16. Given that $x = 1$ is a root of the polynomial $2x^3 - x^2 - 7x + 6 = 0$, find the other two roots. 16. _____
17. Suppose $f(x) = A \log_{10}x + B$ for some constants A and B . If $f(1) = 10$ and $f(10) = 1$, find A and B . 17. _____
18. Use the addition formula $\sin(s + t) = \sin s \cos t + \cos s \sin t$ to find $\sin \frac{7\pi}{12}$. 18. _____
19. A regular 12-sided polygon is inscribed in a circle of radius r . Find the area of the polygon. 19. _____
20. There are 50 new cars on a certain car lot. Suppose 30 of the cars have bucket seats; 27 have a radio; 22 have air-conditioning; 15 have bucket seats and a radio; 10 have air-conditioning and a radio; 12 have bucket seats and air-conditioning; and 5 of the cars have all three options. How many cars on the lot have none of the options? 20. _____

Senior Examination - 1983
(Section II)

NAME _____ SCHOOL _____

1. Give the radian measure of an angle with degree measure 72° . 1. _____
2. Find all points (x, y) which lie on both the parabola $x^2 - y = 1$ and the straight line $2x - y + 3 = 0$. 2. _____
3. Express the decimal number 153 as a binary number. 3. _____
4. Given that $\log_b 2 = 1.2$ and $\log_b 3 = 1.9$, find $\log_b(18b^3)$. 4. _____
5. Consider the function f whose graph is sketched below. List the letters of all those of the following statements which are true.

- a) $f(2) \geq f(x)$ for all x
 - b) $f(x) < 0$ if $-3 < x < 1$
 - c) $f(-1) < f(0)$
 - d) $f(-2) = -f(2)$
 - e) $f(x) < 0$ only if $-3 < x < 1$

5. _____
6. A circle has a radius 10. Find the radius of a circle which has twice the area of the given circle. 6. _____
7. Given that $f(x) = 3x^2 - 2x + 2$, and $h \neq 0$, find $\frac{f(x+h) - f(x)}{h}$ and simplify. 7. _____
8. Find all values of x in the interval $[0, 2\pi)$ which satisfy the equation $2\sin^2 x - \sin x = 1$. 8. _____
9. Determine all values of y such that the points $(-1, 4)$, $(2, 1)$, and $(0, y)$ are the vertices of a right triangle with right angle at $(0, y)$. 9. _____
10. Find the length of a diagonal of a cube which has volume 8. 10. _____

(OVER)

11. In chemistry, the concentration of hydrogen ions in a given substance is denoted H^+ and is measured in molecules per liter. The pH of the substance is defined to be the number $pH = -\log_{10} H^+$. Given that the hydrogen ion concentration of beer is $H^+ = .8 \times 10^{-4}$ molecules per liter, which of the following is the pH of beer?
 a) $4 \log_{10}(.8)$ b) $5 - \log_{10} 8$ c) $4 - \log_{10} .8$ d) $5 - \log_{10}(-8)$

11. _____

12. Determine the amplitude, period, and phase shift of the function $f(x) = -3\sin(2x - \pi)$.

12. amp.= _____
 period= _____
 phase shift= _____

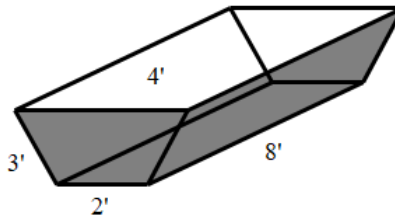
13. Find an equation of the circle having center $(2,2)$ and passing through the point $(0,0)$. Express your answer in the form $(x - a)^2 + (y - b)^2 = r^2$.

13. _____

14. Determine all values of b so that the polynomial $x^2 + bx + 25$ has exactly one real root.

14. _____

15. A water trough has ends which are isosceles trapezoids with sides as shown; its sides and bottom are rectangles. Find the volume of the trough in cubic feet.



15. _____

16. Given that $f(x) = \frac{2x+5}{3x-4}$, determine the value of $f^{-1}(1)$, where f^{-1} denotes the inverse function of f .

16. _____

17. If the solutions to $f(x) = 0$ are -1 and 2 , find the solutions to $f(\frac{x}{2}) = 0$.

17. _____

18. A set of 4 tubes in a radio consists of two good tubes and two defective ones. If 3 tubes are selected at random from this group, what is the probability that exactly two of the three selected will be defective?

18. _____

19. Find the smallest and largest of the following three numbers: $\sin 3$, $\cos 3$, $\csc 3$.

19. smallest _____
 largest _____

20. The diagonals AC and BD of quadrilateral $ABCD$ are perpendicular. If $\overline{AB} = 2$, $\overline{BC} = 3$, and $\overline{CD} = 4$, find \overline{DA} .

20. _____