

# Luzerne County Mathematics Contest

Luzerne County Council of Teachers of Mathematics

Wilkes University - 1984 Junior Examination

(Section 1)

NAME: \_\_\_\_\_ ADDRESS: \_\_\_\_\_

SCHOOL: \_\_\_\_\_ CITY/ZIP: \_\_\_\_\_

TELEPHONE: \_\_\_\_\_

**Directions:** For each problem write your answer in the space provided. Do not use decimal approximations for  $\pi$ ,  $\sqrt{2}$ , etc. Simplify all answers.

- 1) Find all values of  $x$  such that

$$\frac{2x}{x+2} - 1 = \frac{2x+4}{x^2-4}$$

1) \_\_\_\_\_

- 2) Determine  $k$  so that the line  $6x + ky - 7 = 0$  has slope 3.

2) \_\_\_\_\_

- 3) An isosceles right triangle has hypotenuse of length 2. Find the length of one of its legs.

3) \_\_\_\_\_

- 4) Solve for  $y$  if

$$3x + y = 4$$

$$3x - y = 0$$

4) \_\_\_\_\_

- 5) If  $f$  is a function such that

$$f(x) = 3 - 2x, (3 - 2x).$$

5) \_\_\_\_\_

- 6) Find all values of  $x$  such that  $|2x + 7| < 5$ .

6) \_\_\_\_\_

- 7) A circle has a circumference of 10 in. Find its area in sq. in.

7) \_\_\_\_\_

- 8) Solve for  $y$  in terms of  $x$  :

$$x^2y - x + y = 1$$

8) \_\_\_\_\_

- 9) Find all values of  $x$  such that

$$2^{|x-5|} = 8$$

9) \_\_\_\_\_

- 10) The hypotenuse of a right triangle has length 4 in. Find the area of the triangle in sq. in.

10) \_\_\_\_\_

11) If  $\sin \alpha = \frac{5}{13}$  where  $0 < \alpha < \frac{\pi}{2}$ , find  $\tan \alpha$ . 11) \_\_\_\_\_

12) Find the coordinates of the points of the intersection of the line  $y = 2x$  and the parabola  $x^2 - 2x - y = 0$ . 12) \_\_\_\_\_

13) A triangle has an area of 10. sq in. If each of its sides is doubled in length, what is the area of the resulting triangle? 13) \_\_\_\_\_

14) Find A and B such that

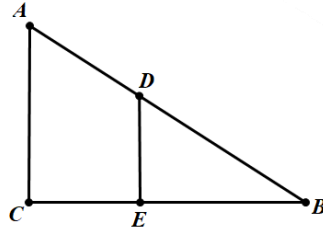
$$\frac{A}{x+3} + \frac{B}{x-2} = \frac{6x+8}{x^2+x-6}$$
 14) \_\_\_\_\_

is an identity.

15) Find  $\sin(\text{Arccos}(\frac{-2}{3}))$ . 15) \_\_\_\_\_

16)  $x = 3$  is a solution to the equation  $x^3 - 10x + 3 = 0$ . Find the other solutions. 16) \_\_\_\_\_

17) Triangle ABC is a right triangle with right angle at C.  $DE \perp BC$ .  $\overline{AC} = 4$ ,  $\overline{DE} = 2$ ,  $\overline{EB} = 3$ . Find  $\overline{AB}$ , the length of AB.

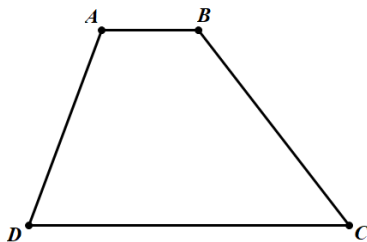


17) \_\_\_\_\_

18) A tin can in the shape of a right circular cylinder has volume  $100\pi$  cu. In. Express the total surface area A of the can (including lids) as a function of the radius, r, of the lids. 18) \_\_\_\_\_

19) Solve for x if  $\log_3(\frac{1}{x+1}) + \log_3(x+4) = 1$ . 19) \_\_\_\_\_

20) ABCD is a trapezoid with  $\overline{AB} = 1$ ,  $\overline{BC} = 3$ ,  $\overline{CD} = 4$ , and  $\overline{DA} = 2$ . Find the area of the trapezoid.



20) \_\_\_\_\_

# Luzerne County Mathematics Contest

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(Section 2)

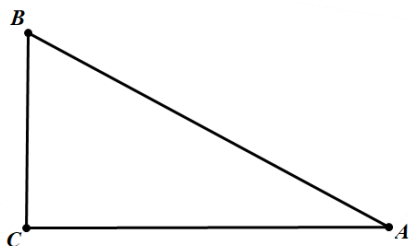
NAME: \_\_\_\_\_ ADDRESS: \_\_\_\_\_

SCHOOL: \_\_\_\_\_ CITY/ZIP: \_\_\_\_\_

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**Directions:** For each problem write your answer in the space provided. Do not use decimal approximations for  $\pi$ ,  $\sqrt{2}$ , etc. Simplify all answers.

- 1) Find all values of  $x$  such that  $x(x - 1) = 1$ . 1) \_\_\_\_\_
- 2) In triangle  $ABC$ ,  $\overline{AB} = 13$ ,  $\overline{AC} = 12$ , and  $\overline{BC} = 5$ . Find the length of the altitude to side  $AB$ . 2) \_\_\_\_\_



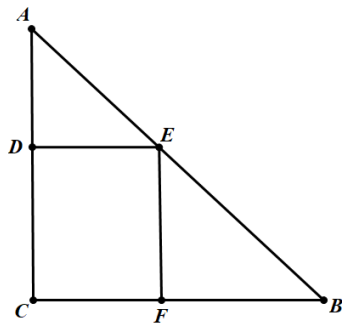
- 3) Find the coordinates of the  $x$ -intercepts of the parabola  $y = 2x^2 - 12x + 16$ . 3) \_\_\_\_\_
- 4) Solve for  $x$  in terms of  $y$  if  $x^2y - x + y = 1$ . 4) \_\_\_\_\_
- 5) Find all values of  $x$  in the interval  $[0, 2\pi]$  for which  $\sin x > \cos x$ . 5) \_\_\_\_\_
- 6) Find all values of  $x$  such that  $1 + \sqrt{2x + 6} = \sqrt{4x + 5}$ . 6) \_\_\_\_\_
- 7) A bicycle tire has a diameter of 26 in. If the bicycle is ridden for 1 mile, which of the following numbers is the best approximation of the number of revolutions the tire will make? 7) \_\_\_\_\_
- (a) 2500      (b) 1250
- (b) (c) 880      (d) 1700 8) \_\_\_\_\_
- 8) Find all values of  $x$  such that  $x^2 * 2^x - 3x * 2^x = 0$ . 9) \_\_\_\_\_
- 9) If  $\log_b a = k$  and  $c = a^2$  find  $\log_b c$  in terms of  $k$ . 9) \_\_\_\_\_
- 10) Find the coordinates of the vertex of the parabola  $x^2 + 4x - y = 0$ . 10) \_\_\_\_\_

11) Let  $f$  be a function defined by  $f(x) = 3x - 4$ . Suppose  $g$  is a function such that  $f(g(x)) = x$  for all  $x$ . Find  $g(x)$ . 11) \_\_\_\_\_

12) Which of the following is the best approximation (in degrees) of 1 radian? 12) \_\_\_\_\_  
 (a)  $1^\circ$  (b)  $30^\circ$  (c)  $60^\circ$  (d)  $90^\circ$

13) If  $|x - 3| < 1$ , find the smallest value of  $a$  such that  $|2x + 3| < a$ . 13) \_\_\_\_\_

14) Triangle  $ABC$  has a right angle at  $C$ .  $\overline{DEFC}$  is a rectangle.  $\overline{AD} = 15$ ,  $\overline{DE} = 20$ ,  $\overline{EF} = 30$ . Find the length of  $AB$ . 14) \_\_\_\_\_



15) Solve for  $x$  if  $x^{\sqrt{\log x}} = 10^8$ . 15) \_\_\_\_\_

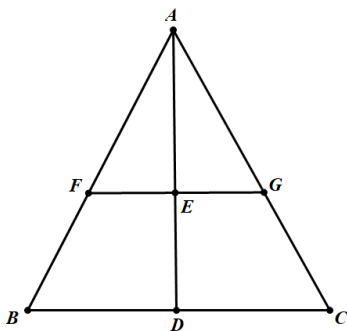
16) If we divide  $x^3 - 3x^2 + 2x + 4$  by  $x^2 - 2$ , we find that  $\frac{x^3 - 3x^2 + 2x + 4}{x^2 - 2} = x - 3 + \frac{f(x)}{x^2 - 2}$ . Find  $f(x)$ . 16) \_\_\_\_\_

17) Suppose  $f(x) = ax^2 + 2x + 5$ . Determine  $a$  so that  $f(1) = f(-2)$ . 17) \_\_\_\_\_

18) A rectangular box (with lids) has a square base, a height of 8 in. and a volume of 400 cu. in. Find the total surface area of the box. 18) \_\_\_\_\_

19) Find all values of  $x$  in the interval  $[0, 2\pi]$  such that  $\cos\left(\frac{x}{2}\right) = \frac{1}{2}$ . 19) \_\_\_\_\_

20) Triangle  $ABC$  is equilateral with sides of length 2 in.  $FG \parallel BC$  and  $AD \perp BC$ . If  $\overline{AE} = 1$  in., find the area of  $\triangle AFG$ . 20) \_\_\_\_\_



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**Directions:** For each problem write your answer in the space provided. Do not use decimal approximations for  $\pi$ ,  $\sqrt{2}$ , etc. Simplify all answers.

- 1) Find all values of  $x$  such that  $\frac{2x}{x-2} - 1 = \frac{2x+4}{x^2-4}$ . 1) \_\_\_\_\_
- 2) Determine  $k$  so that the line  $6x + ky - 7 = 0$  is perpendicular to the line  $3x - 2y + 4 = 0$ . 2) \_\_\_\_\_
- 3) An isosceles right triangle has hypotenuse of length 2 in. Find the area of the triangle in sq. in. 3) \_\_\_\_\_
- 4) Find all values of  $x$  in the interval  $[0, 2\pi]$  such that  $2 \sin x - 1 = 0$ . 4) \_\_\_\_\_
- 5) Find all values of  $x$  such that  $|7 - 2x| < 5$ . 5) \_\_\_\_\_
- 6) The graph of the equation  $x^2 + y^2 - 4x + 6y = 4$  is a circle. Find the coordinates of its center. 6) \_\_\_\_\_
- 7) Express the area  $A$  of a circle as a function of its circumference  $C$ . 7) \_\_\_\_\_
- 8) Solve for  $y$  in terms of  $x$ :  
 $\frac{x-y}{x} = x + y$ . 8) \_\_\_\_\_
- 9) Find all values of  $x$  such that  $2^{|x^2+3x-1|} = 8$ . 9) \_\_\_\_\_
- 10) The hypotenuse of a right triangle has length  $2\sqrt{5}$  in. and one leg has length 4 in. Find the area of the triangle in sq. in. 10) \_\_\_\_\_

11) If  $\sin \alpha = \frac{5}{13}$  where  $\pi/2 < \alpha < \pi$ , find  $\tan \alpha$ .

11) \_\_\_\_\_

12) Find the coordinates of the points of intersection of the line  $y = 2x$  and the parabola  $x^2 - 2x - y = 0$ .

12) \_\_\_\_\_

13) Which of the following numbers is the best approximation of  $\sin 1$ ?

(a) -0.5 (b) 0

(c) 0.5 (d) 0.8 (e) 1.0

13) \_\_\_\_\_

14) Find A and B so that  $\frac{A}{x+3} + \frac{B}{x-2} = \frac{6x+8}{x^2+x-6}$  is an identity.

14) \_\_\_\_\_

15) Find, in terms of x,  $\sin(\text{Arccos } x)$ .

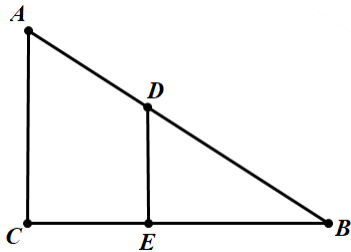
15) \_\_\_\_\_

16) Find all values of x such that  $x^3 - 10x + 3 = 0$ .

16) \_\_\_\_\_

17) Triangle ABC is a right triangle with right angle at C.  $DE \perp BC$ .  $\overline{AC} = 4$ ,  $\overline{DE} = 2$ , and  $\overline{EB} = 3$ . Find the length of AB.

17) \_\_\_\_\_



18) A tin can in the shape of a right circular cylinder has volume  $100\pi$  cu. in. Express the total surface area A of the can (including the lids) as a function of the radius r of its lids.

18) \_\_\_\_\_

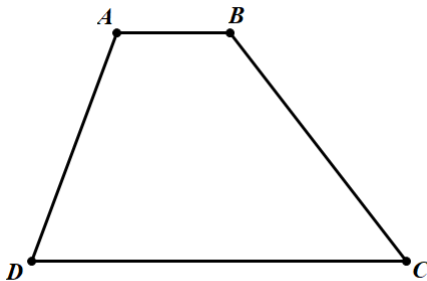
19) Solve for x if

$$\log_3 \left( \frac{1}{x+1} \right) = 1 - \log_3(x+4).$$

19) \_\_\_\_\_

20) ABCD is a trapezoid with  $\overline{AB} = 1$ ,  $\overline{BC} = 3$ ,  $\overline{CD} = 4$ , and  $\overline{DA} = 2$ . Find the area of the trapezoid.

20) \_\_\_\_\_



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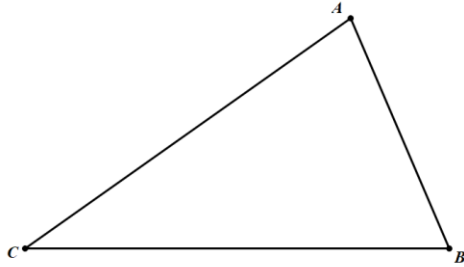
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**Directions:** For each problem write your answer in the space provided. Do not use decimal approximations for  $\pi$ ,  $\sqrt{2}$ , etc. Simplify all answers.

- 1) Find all values of  $x$  such that  $x(x - 1) = 1$ .
- 2) In triangle ABC,  $\overline{AB} = 2$ ,  $\overline{AC} = 4$ , and  $\overline{BC} = 4$ . Find the length of the altitude to side BC.



1) \_\_\_\_\_

2) \_\_\_\_\_

- 3) Find the coordinates of the x-intercepts of the parabola  $y = 2x^2 - 12x + 16$ .

3) \_\_\_\_\_

- 4) Solve for  $x$  in terms of  $y$  if  $\frac{x-y}{x} = x + y$ .

4) \_\_\_\_\_

- 5) Find all values of  $x$  in the interval  $[0, 2\pi]$  for which  $|\sin x| > |\cos x|$ .

5) \_\_\_\_\_

- 6) Find all values of  $x$  such that

$$\sqrt{4x + 5} - \sqrt{2x + 6} = 1.$$

6) \_\_\_\_\_

- 7) Find  $\cos(2 \operatorname{Arcsin} \frac{3}{5})$ .

7) \_\_\_\_\_

- 8) Find all values of  $x$  such that  $x^2 \cdot 2^x - 3x \cdot 2^x = 0$ .

8) \_\_\_\_\_

- 9) If  $\log_b a + \log_b c = m$ , find  $\log_b (ac)^2$  in terms of  $m$ .

9) \_\_\_\_\_

- 10) Find the coordinates of the vertex of the parabola

$$x^2 + 4x + y = 0.$$

10) \_\_\_\_\_

11) If  $f$  is a function such that  $f(x + y) = f(x) \cdot f(y)$  for all real numbers  $x$  and  $y$ , and  $f(1) = 3$ , find  $f(3)$ .

11) \_\_\_\_\_

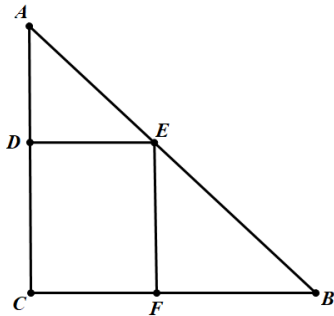
12) If  $f(x) = \frac{\cos x}{x}$  which of the following is the best approximation of  $f(100)$ ?

12) \_\_\_\_\_

13) If  $|x - 3| < 1$ , find the smallest value of  $a$  such that  $|2x + 3| < a$ .

13) \_\_\_\_\_

14) Triangle  $ABC$  has a right angle at  $C$ .  $\overline{DEFC}$  is a rectangle.  $\overline{AD} = 15$ ,  $\overline{DE} = 20$ ,  $\overline{EF} = 30$ . Find the length of  $AB$ .



14) \_\_\_\_\_

15) Solve for  $x$  if  $x^{\sqrt{\log x}} = 10^8$ .

16) If we divide  $x^3 - 3x^2 + 2x + 4$  by  $x^2 - 2$ , we find that

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 Find  $f(x)$ .

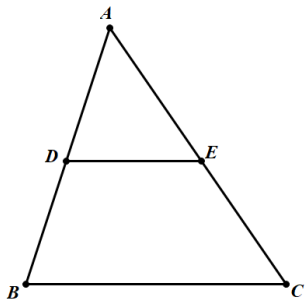
15) \_\_\_\_\_

17) Suppose  $f(x) = ax^2 + 2x + 5$ . Determine  $a$  so that  $f(1) = f(-2)$ .

16) \_\_\_\_\_

18) In triangle  $ABC$ ,  $DE \parallel BC$ ,  $\overline{AD} = 5$  and  $\overline{DB} = 6$ . Find the ratio of the area of  $\triangle ADE$  to the area of the trapezoid  $DECB$ .

17) \_\_\_\_\_



18) \_\_\_\_\_

19) Find all values of  $x$  in the interval  $[0, 2\pi]$  such that

$$\cos\left(\frac{x}{2}\right) = \frac{1}{2}$$

20) A circle is tangent to the line  $y = 2x + 2$  at the point  $(2, 6)$ , and has its center on the  $x$  axis. Find the radius of the circle.

19) \_\_\_\_\_

20) \_\_\_\_\_



