LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes College - - 1985 Junior Examination

(Section I)

NAME:_____   **SCHOOL:**_____

Directions: For each problem write your answer in the space provided. Do not use decimal approximation for π, $\sqrt{2}$, etc. Simplify all answer.

1. Find all values of x such that $x^2 = 8x$.   1. _____

2. Find an equation of the line through the
   point (-5,6) and perpendicular to the x axis.   2. _____

3. If $f(x) = \frac{x+8}{x-2}$ what is f (f (7))?   3. _____

4. ΔABC is a right triangle with right angle at C.
   If $\overline{AB}$ = 17 and $\overline{BC}$ + 15. Find $\overline{AC}$.   4. _____

5. Determine k so that the point (3,-2) is on the
   line kx – 2y + 7 = 0.   5. _____

6. Suppose $0 \leq \theta \leq \pi$ and $\cos\theta = \frac{24}{25}$. What is $\sin\theta$.   6. _____

7. Find the center and radius of the circle $x^2 + y^2 - 2y = 3$.   7. center _____

   radius _____

8. In ABC, $\overline{AB} = 7$, $\overline{AC} = 7$ and $\overline{BC} = 8$. Find the length
   of the altitude to side AC.   8. _____

9. Find all ordered pairs (x , y) satisfying both of the
   equations   4x – 3y = 2

   7x + y = 6   9. _____

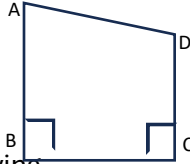10. Solve for x in the terms of y : $2y = \frac{x-2}{3x-1}$ 10. _____

(OVER)

11. If f is a linear function such that f (-1) = 8

and f (2) = 5, find f (x).   11. _____

12. In quadrilateral ABCD, $AB \perp BC$ and $DC \perp BC$.

$\overline{AB} = 4, \overline{DC} = 3, and \overline{BC} = 2.$

Find the area of the quadrilateral.   12._____

13. Find all x in the interval [0, 2π] satisfying

$2 \sin^2 x - \sin x - 1 = 0.$ 13. _____

14. Give an equation of the ellipse centered at the

origin with x-intercepts ± 7 and y-intercepts ± 2.   14. _____

15. Given that x = $-\frac{1}{2}$ is a solution t the equation

$2x^3 - 7x^2 + 2x + 3 = 0$, find all other solutions.   15. _____

16. $\triangle ABC$ is inscribed in a circle, with AB as a diameter.

If the radius of the circle is 5, express the area A of the

triangle as a function of x, where x is the length of side BC.   16. A(x)=_____

17. Find all x such that $\frac{4}{x} < \frac{3}{5}$.   17. _____

18. Two trains leave the same city at the same time traveling

at constant speeds. One train, going north, travels 20 mph

faster than the other, which is going east. If they are 300 miles

apart after 5 hours, what is the speed of the northbound train in

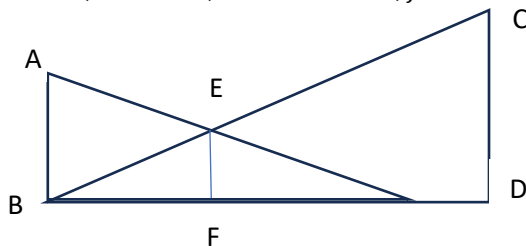mph to the nearest multiple of 10?   18. _____

19. The number $2^{48} - 1$ is exactly divisible by two numbers between

60 and 70. What are these two numbers?   19. _____

20. Given that AB, EF, and CD are perpendicular to BD and that

$\overline{AB} = 10, \overline{BD} = 30, and \overline{CD} = 20, find \overline{EF}.$ 20. _____

(OVER)

Junior Examination, 1985

(Section II)

NAME:_____   **SCHOOL:**_____
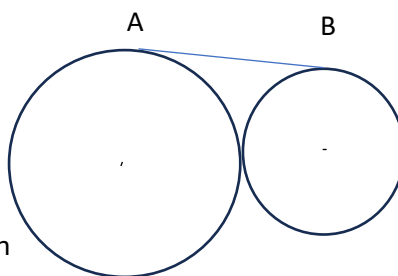
1. Find all x such that $\frac{|x+2|}{5} = 3$.

1. _____

2. Find the coordinates of the vertex

   of the parabola x = 2 (y -2)² +1.

2. _____

3. Two circles are tangent are shown.

   The larger circle has radius 7 and

   the smaller circle has radius 5. AB

   is a common tangent. Find the length

   of AB.

3. _____

4. If $\cos \theta = \frac{2}{3}$, find cos (- θ).

4. _____

5. Find all x such that $\sqrt{x+7} + 5 = x$.

5. _____

6. Determine all x such that $2^{2x+1} = \frac{1}{8}$.
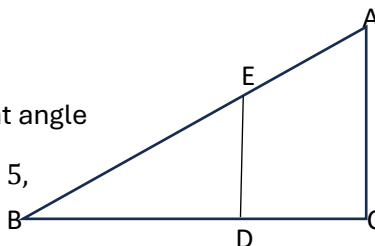
6. _____

7. Express as a single logarithm:

   3 log x + log (x – 1) -2 log (x +1).

7. _____

8. Find Arctan ($\tan \frac{3\pi}{4}$).

8. _____

9. $\triangle ABC$ is a right triangle with right angle

   at C . DE ⊥ BC. If $\overline{ED} = 8, \overline{DC} = 5$,

   and $\overline{BD} = 10, find \overline{AB}$.

9. _____

10. Suppose f (x) = 3 x² – x and h ≠ 0.

    Find $\frac{f(x+h) - f(x)}{h}$ and simplify.

10. _____

(Over)

11. Find x if the point (x , -4) lies on the line

which passes through the points (0 , 8) and (-4 , 0).   11. _____

12. Find the radius of the circle circumscribed

about an equilateral triangle having sides of

length 10.   12. _____

13. Find all x in the interval [0, π] satisfying

2 cos 2x = 1.   13. _____

14. Which is smallest?

(a) 3 + $\sqrt[3]{25}$ (b) $\sqrt[3]{200}$

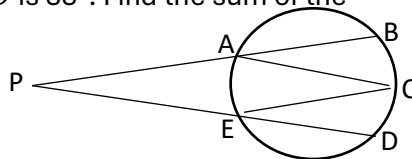(c) 2 + $\sqrt{17}$ (d) 2π 14. _____

15. If x = $\log_8$ 225 and y = $\log_2$ 15, find x in terms of y.   15. _____

16. A square has sides of length s. A circle has area twice the

area of the square. Express the radius of the circle in terms of s.   16. _____

17. Find the center and radius of the circle which

passes through the points (5, 3), (-2, -4), and (2, -6).   17. center_____

radius_____

18. Find the radius of the inscribed circle of ΔABC

if $\overline{AB} = 6, \overline{BC} = 7 \; and \; \overline{CA} = 5.$ 18. _____

19. Points A, B, C, D and E are on a circle as shown. The measure of

arc $\widehat{BC}$ is 42° and the measure of arc $\widehat{CD}$ is 38°. Find the sum of the

measures of angles P and C.   19._____

20. Towns A and B are connected by a highway. A truck leaves town A

headed for town B at a constant speed. At the same moment a car leaves

town B headed for town A at a constant speed. After they pass each other

at a point between A and B, it takes the truck 2$\frac{1}{4}$ hours to complete its trip

and it takes the car 1 hour to complete its trip. What is the total time

(in hours) of the truck's trip?   20._____

(OVER)

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes College - - 1985 Senior Examination

(Section I)

NAME:_____   SCHOOL:_____

Directions: For each problem, write your answer in the space provided. Do not use

approximations. Simplify all fractions and radicals, and rationalize

denominators. Your answer must be complete to receive credit for a problem.

1. Find all values of x such that $2x^2 + 5x + 2 = 0$.   1. _____

2. Find an equation of the line through the point (-5, 6) and

    perpendicular to the line x + 3y = 7.   2. _____

3. If f (x) = $\frac{x+8}{x-2}$, $what\ is\ f(f(x))$?   3. _____

4. Find the measure (in degrees) of an interior angle of a

    rectangle pentagon.   4. _____

5. Determine k so that the point (3, -2) is on the line

    kx -2y + 7 = 0.   5. _____

6. Suppose $-\pi \leq \theta \leq \pi$ and cos $\theta = \frac{24}{25}$. What is sin θ?   6._____

7. Find the center and radius of the circle $3x^2 - 9x + 3y^2 + 6y = 0$.   7. _____

8. In ΔABC , $\overline{AB} = 7, \overline{AC} = 7, and\ \overline{BC} = 8$. Find the length of the

    Altitude of side AC.   8. _____

9. Find all ordered pairs (x, y) satisfying both of the equations

    $X^2 -2x -y =1$

       5x -y =13   9. _____

10. Solve for x in terms of y:
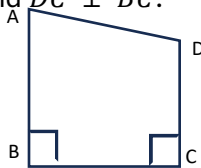
    $y = \frac{2x-5}{x+3}$ 10. _____

(Over)

11. A chemical solution is heated in such a way that the

temperature T is a linear function of time t. If the initial

temperature is 80° and the temperature after 20 minutes

is 90°, find a general expression for the temperature T(t)

after t minutes.   11. T(t)=_____

12. In quadrilateral ABCD, $AB \perp BC$ and $DC \perp BC$.

$\overline{AB} = 4, \overline{DC} = 3, and\ \overline{BC} = 2.$

Find the area of the quadrilateral.   12. _____

13. Find all x in the interval [0, 2 π] satisfying $\sin^2 x - 3\cos^2 x = 0$.   13. _____

14. Suppose an ellipse is centered at (2, 3) and the ends of the

axes are the points (2, -1), (0, 3), (2,7), and (4,3). Give an

equation of the ellipse.   14. _____

15. Find all x such that $2x^3 - 7x^2 + 2x + 3 + 0.$ 15. _____

16. A rectangular box has square bases with side of length x

and a volume of 100 cu. in. Express the total surface area A

of the box (including bases) as a function of x.   16. A(x)=_____sq.in.

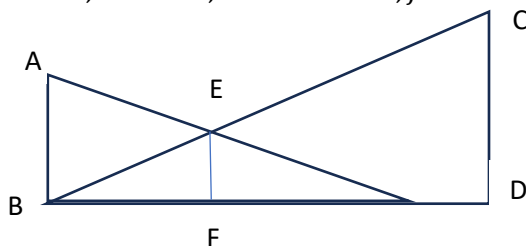17. Find all x such that $\frac{x-1}{x} < 4$.   17. _____

18. Two trains leave the same city at the same time traveling

constant speeds. One train going north, travels 20 mph faster

than the other, which is going east. If they are 300 miles apart

after 5 hours, what is the speed of the northbound train in mph

to the nearest multiple of 10?   18. _____

19. If the line y=mx +1 intercepts the ellipse $x^2 + 4y^2 = 1$ exactly once,

what is the value of m^2?   19. m^2=_____

20. Given that AB, EF, and CD are perpendicular to BD and that

$\overline{AB} = 10, \overline{BD} = 30,\ and\ \overline{CD} = 20, find\ \overline{EF}.$ 20. _____

(OVER)

1985 Senior Examination

(Section II)

NAME:_____   **SCHOOL:**_____

1. Find all x such that $\left|\dfrac{x+2}{x+6}\right| = 3$ 1._____

2. Find the coordinates of the vertex of the parabola

   X=2y²-8y+9   2. _____

3.A track is in the shape of a rectangle with semicircles at

  Two opposite sides. If the perimeter of the track is 400

  Meters, express the total area A of the figure as a function

  of r, the radius of the semicircles.   3. _____

4.Find all x in the interval $[0,2\pi]$ satisfying 2 cos 2x=1.   4. _____

5.Find all x such that $\sqrt{x-3} + \sqrt{x+5} = 4$.   5. _____

6.Determine x such that $8^{x+2}=4^{3x-1}$ 6. _____

7. What is the domain of the function f(x) = x log (-x + 3) – 10?   7. _____

8. A square and a circle have equal perimeters. Find the

  ratio of the area of the circle to the area of the square.   8. _____

9.Find x if the point (x, -4) lies on the line which passes

  through the points (0, 8) and (-4, 0).   9. _____

10. Which of the following is the best approximation of

  $\dfrac{\pi^2}{\cos 1}$. (a) 100   (b) 10 (c) 20 (d) 0   10. _____

                                                               (OVER)

11. Suppose $f(x)=\dfrac{2}{x+1}$ and $h\neq0$. Determine $\dfrac{f(x+h)-f(x)}{h}$ and simplify. 11. _____

12. Find the radius of the circle circumscribed about $\triangle ABC$ if $\overline{AB}=5$,

$\overline{AC}=5, and\ \overline{BC}=9$. 12. _____

13. Suppose $-\dfrac{\pi}{2}\le\theta\le\dfrac{\pi}{2}$ and $\sin\theta$=a. Express sin 2θ in terms of a. 13. _____

14. Which is smallest?

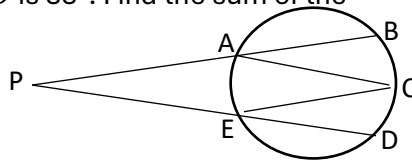(a) $3+\sqrt[3]{25}$ (b) $\sqrt[3]{200}$

(c) $2+\sqrt{17}$ (d) 2π 14. _____

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17. Find the center and radius of the circle which

passes through the points (5, 3), (-2, -4), and (2, -6). 17. center_____

radius_____

18. Find the radius of the inscribed circle of $\triangle ABC$

if $\overline{AB}=6, \overline{BC}=7\ and\ \overline{CA}=5$. 18. _____

19. Points A, B, C, D and E are on a circle as shown. The measure of

arc $\widehat{BC}$ is 42° and the measure of arc $\widehat{CD}$ is 38°. Find the sum of the

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headed for town B at a constant speed. At the same moment a car leaves

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at a point between A and B, it takes the truck $2\dfrac{1}{4}$ hours to complete its trip

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