LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes College - - 1989 Junior Examination
(Section I)

NAME:_____ **SCHOOL**:_____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals, and rationalize denominators. Your answer must be complete to receive credit for a problem.

1. Harry had grades of 70, 80, 85, and 80 on his quizzes. If all his quizzes have the same weight, what grade must he get on his last quiz so that his average will be 80? 1. _____

2. A circle has radius 10. Find the radius of a circle which has twice the area of the given circle. 2. _____

3. If $\pi/2 < t < \pi \; and \; \sin t = 5/13$, then find cos t. 3. _____

4. Two fair 6-sided dice are thrown. What is the probability that the sum of the dice faces is 9? 4. _____

5. If $x^{logx} = 100x$, find all possible values for x. (Note: log x is logarithm base 10 of x) 5. _____

6. Simplify, leaving no negative exponents:
$(x^{-1} + y^{-1})(x + y)^{-1}$ 6. _____

7. Solve for x : $\frac{1}{x+1} > \frac{3}{2}$ 7. _____

8. Give a formula for $f^{-1}(x) \; if \; f(x) = 1 - 2x^3$. 8. _____

9. If $f(x) = \sqrt{x} \; and \; g(x) = x^2 - 5x + 6$, find $(f \circ g)\,(1)$. 9. _____

10. In problem 9 above, what is the domain of $f \circ g$? 10. _____

(OVER)

11. Find a quadratic polynomial function whose graph passes through the points (1,0), (-1,0), (0,5). 11. _____

12. Solve for y : $\sqrt{6y+7} - \sqrt{3y+3} = 1$ 12. _____

13. How many distinct divisors does the integer 18000 have (including 1 and 18000 itself)? Find the sum of These distinct divisors. 13. Number_____

Sum_____

14. The line $4x + 3y = 12$ forms a triangle with the x and y-axes. If this triangle is rotated about the y-axis, a right circular cone is formed. Find the volume of this cone. 14. _____

15. Find an equation for the line through (-1,5) which is perpendicular to the line $2x + y + 4 = 0$. 15. _____

16. Find the two points in the plane equidistant from $y = x, y = -x$ and the point (5,3). 16. _____

17. Find: $\sin\frac{11\pi}{12}$ 17. _____

18. What is the center and radius of the circle $3x^2 + 3y^2 - 18x + 6y = -2$? 18. Center_____

Radius_____

19. A piece of wire c inches in length is to be cut into two pieces. One will be shaped into a circle, the other into a square. Express the total area in terms of the radius of the circle. 19. _____

20. If f(x) is a polynomial of degree 5 with real coefficients and if f(1) = f(3) = 0, find a quadratic polynomial that divides f(x) evenly. 20. _____

1989 Junior Examination
(Section II)

NAME:_____ **SCHOOL**:_____

1. The square of twice a number is larger than the square of the sum of the number and 1. Which number possess this property? 1. _____

2. What is the value of $\log_8 16$? 2. _____

3. In a group of 30 students, 20 take French, 15 take Spanish, and 5 take neither. How many students take both French and Spanish? 3. _____

4. Simplify, leaving no negative exponents: $\frac{(5m^2 2)(3mn^2)^2}{9^{-1}m^2(n^2)^{-2}}$ 4. _____

5. Solve for $x\ in\ [0,2\pi) : \sin x > -\frac{1}{2}$. 5. _____

6. A rectangular box has edges in the ratio 1 : 2 : 3 . How much must the smallest edge (initially length 10), be increased by, in order for the volume to be increased by 10% ? 6. _____

7. For what value(s) of x does the graph of $f(x) = \frac{x-3}{x^2-9}$ have a vertical asymptote? 7. _____

8. Find all values of x such that $|x - 7| \geq 9$? 8. _____

9. What is the distance between the points (6,5) and (-3,-4)? 9. _____

10. A man standing 16 feet from an 18-foot light pole casts a shadow From the light 8 feet long. How tall is he? 10. _____

(OVER)

11. Find $\cos(\arcsin x)\ for -1 \leq x \leq 1.$ 11. _____

12. What is the next term in the geometric progression
$\sqrt[4]{3}, \sqrt{3}, \sqrt[4]{27}$? 12. _____

13. A circle is circumscribed about a square which has sides of Length 4, and another circle is inscribed in the same square. Find the area of the region between the two circles. 13. _____

14. In a certain class, the average weight for the girls was 128 pounds, the average for boys was 160 pounds, and The class as a whole 146 pounds. How many people Are in the class, given that there are 14 girls? 14. _____

15. What is the y-intercept of the line $2x - 3y + 5 = 0$? 15. _____

16. For what values of $x\ is\ \left|\frac{x+3}{x-1}\right| \leq 2$? 16. _____

17. Which of the following lines **do not** pass through the third quadrant:

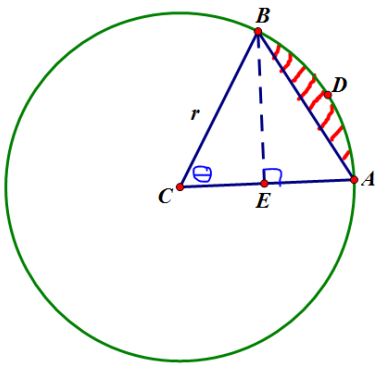
a) y = x
b) y = 4x – 7
c) y = -2x - 5
d) y = 4x + 7
e) y = -2x + 5

17. _____

18. Arrange the following numbers from smallest to largest: 18. _____
$222^2, 22^{22}, 2^{222}$

19. For what value(s) of c will $cx^2 - cx + 3 = 0$ have repeated roots? 19. _____

20. In the figure, the shaded region is a segment ADB of a circle cut off by cord AB. Find the area of the segment in terms of the radius r of the circle and θ, The central angle.



20._____

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes College - - 1989 Senior Examination
(Section I)

NAME:_____ **SCHOOL**:_____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals, and rationalize denominators. Your answer must be complete to receive credit for a problem.

1. A student made grades of 45, 60, and 72 in three tests. What must she score on the final examination, which counts as two tests, in order to attain a passing average of 75? 1._____

2. Find an equation of a circle with center (1,1) which passes through (5,2). 2. _____

3. Find the length of a diagonal of a cube which has volume 8. 3. _____

4. The area of a given circle can be doubled by increasing its radius by 1 unit. Find the radius of the circle. 4. _____

5. If $3x + 4 = 2(y + 2)$, what is the ratio of y : x ? 5. _____

6. Find the intersection point(s) between the line $2x + y = 6$ and the parabola $y = 2x^2 - x + 6$. 6. _____

7. Simplify, leaving no negative exponents:
$(x^2 - 4)^{-1/2} - x^2(x^2 - 4)^{-3/2}$ 7. _____

8. Solve for x : $x^2 + 2x - 10 = -5$ 8. _____

9. Solve the equation $\sqrt{5p - 1} = \sqrt{4p}$ 9. _____

10. In the figure, which of the following have the graph pictured?

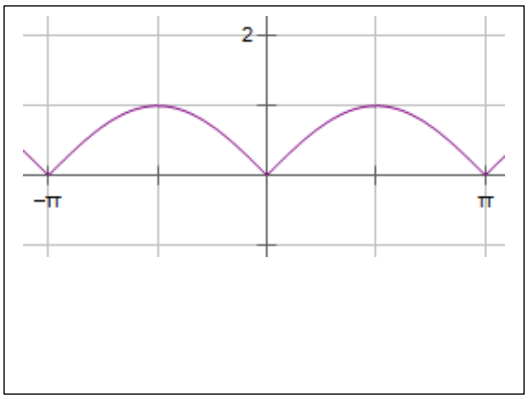
a) $f(x) = \begin{cases} \sin x \ \ if \ x \geq 0 \\ \cos x \ \ if \ x < 0 \end{cases}$

b) $g(x) = \cos x - 1$

c) $h(x) = \begin{cases} \sin(x) \, if \ x \geq 0 \\ \sin(-x) \, if \ x < 0 \end{cases}$

d) $k(x) = |\sin x|$

e) $p(x) = \sqrt{1 - sin^2 x}$



10. _____

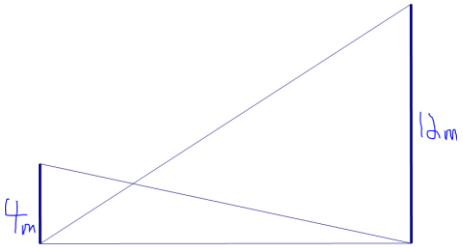
(OVER)

11. Find a quadratic polynomial function whose graph passes through the points (1,0), (-1,0), (2,-15). 11. _____

12. What is the slope of the line through the points (3,-2), and (-4,6) ? 12. _____

13. Find $\csc(Arccos\frac{1}{7})$. 13. _____

14. Two poles are braced as shown. How high above the ground do the braces cross?



14. _____ m

15. Give the coordinates of all points on the line $y = 3$ which are 6 units away from the point (2,-1). 15. _____

16. Find a formula for f(x) if f is a rational function whose graph goes through (2,5) and has exactly the two asymptotes $y = 2x + 3$ and $x = 3$. 16. _____

17. If $(x - 7)$ is a factor of $f(x) = 8x^5 - 52x^4 + 2x^3 - 198x^2 - 86x + 14$, what is another factor? 17. _____

18. What is the period of $f(t) = 4\sin(2t - \frac{\pi}{3})$? 18. _____

19. The width of a rectangular box is 3 times its length, and its height is 2 times its length. Express the volume V of the box in terms of its width, W. 19. _____

20. Suppose a regular polygon of n sides is inscribed in a circle of radius r. If s denotes the length of a single side of the polygon, determine s in terms of r and n. 20. _____

1989 Senior Examination
(Section II)

NAME:_____ **SCHOOL**:_____

1. For what values of t is the following function defined?
   $f(t) = \frac{1}{\sqrt{4-t^2}}$ 1. _____

2. If $x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ where $i = \sqrt{-1}$, find x^3 2. _____

3. Two circles are externally tangent to each other. A tangent line is drawn to the larger circle from the center of the smaller circle. If the tangent is the length 2, and the larger circle has a diameter of 3, what is the radius of the smaller circle? 3. _____

4. Given that $f(x) = x^2 + 2$ *and* $h \neq 0$, find $\frac{f(x+h)-f(x)}{h}$ and simplify. 4. _____

5. A line through (4,4) is tangent to the circle $x^2 + y^2 = 4$ at a point in the fourth quadrant. Find the coordinates of that point. 5. _____

6. Given that for any positive integer n, $1 + 2 + \cdots + n = \frac{n(n-1)}{2}$, Find a formula for $1 + 3 + 5 + \cdots + (2n - 1)$. 6. _____

7. Find the smallest possible value of $f(x, y) = x^2 - 3x + 2y^2 + 4y + 2$, for x, y any real numbers 7. _____

8. Solve for x in $[0,2\pi)$: $\cos x \leq \sin x$. 8. _____

9. Solve for x : $\left|\frac{x+3}{x-1}\right| \leq 2$. 9. _____

10. Simplify, leaving no negative exponents:
    $\left(\frac{4}{49}\right)^{-3/2}$ 10. _____

(OVER)

11. A hiker begins walking up a hill at a spot where the elevation is 0.9km After she has walked 3kms, she sees a sign giving the elevation as 0.95km. If the slope of the hill is constant and she walks in a straight line, how far will she have walked when she reaches an elevation of 1.1 km? 11. _____

12. Simplify: $\frac{2^{20}-2^{17}+7}{2^{17}+1}$ 12. _____

13. Solve for x: $x^{2/3} + 3x^{1/3} + 2 = 0$ 13. _____

14. Is the function $g(x) = \frac{5^x+5^{-x}}{2}$ an odd function, an even function, or neither. 14. _____

15. What is the value of $\log_2 \sqrt{2}$? 15. _____

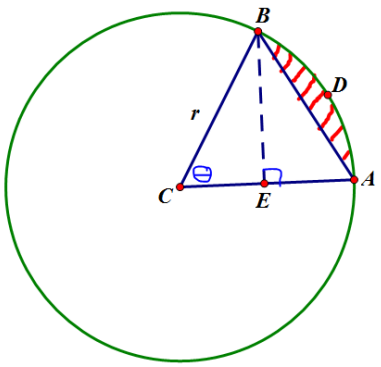
16. Solve simultaneously: $\begin{aligned} x + y - z &= 4 \\ 2x + y + z &= 1 \\ 3x - 2y - z &= 3 \end{aligned}$ 16. _____

17. Specify the solution set for: $\frac{x-3}{x-7} = \frac{3x-17}{x-7}$ 17. _____

18. Two cards are drawn without replacement from a 52-card deck. What is the probability that the first is a queen and the second is a spade? (Recall there are 4 queens, one of each suit, and 13 cards in each of the 4 suits.) 18. _____

19. For what value(s) of c will $cx^2 - cx + 3 = 0$ have repeated roots? 19. _____

20. In the figure, the shaded region is a segment ADB of a circle cut off by cord AB. Find the area of the segment in terms of the radius r of the circle, and the central angle θ.



20._____