LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University - - 1990 Junior Examination
(Section I)

Name: _____ School: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simply all fractions and radicals, and rationalize denominators. Your answers must be complete to receive credit for a problem.

1. Find the equation, in slope-intercept form, of the line that passes through the point (3,2) and its perpendicular to the line $2x - y + 6 = 0$ 1. _____

2. If $3x + 1$ is a factor of $10x^3-2x^2+15x-3$, list a quadratic factor of the expression. 2. _____

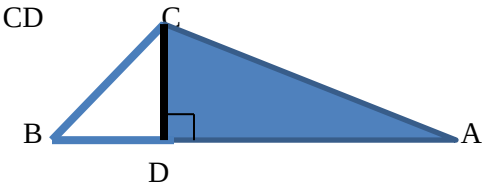
3. Solve for x, y, z:
$x+2y+3z= 4$
$x+ y+ 2z= 5$
$y+ 2z= 4$
3. x= _____
y=_____
z= _____

4. For what real values of z is the function 4. _____

f(x) = $\frac{1}{\sqrt{x^2-x-6}}$ defined

5. Preform the indicated operations and simplify 5. _____
$2x(3x^2 - 6x + 4) - 3x[2x^2 - 4(x + 1)]$

6. In the right triangle ABC, altitude CD Is drawn to hypotenuse AB. If DB= 9 and CD= 15, Find AB. 6. _____



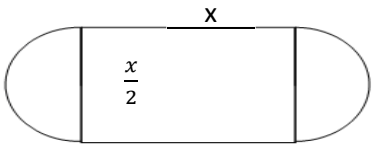
7. Find all real values of x for which $\log_4(3x - 2) = 2$ 7. _____

8. Find $\csc(-\frac{7\pi}{6})$. 8. _____

9. What is the area of a regular hexagon which is inscribed in a circle radius 2. 9. _____

10. The figure below is constructed from a rectangle of the dimensions x and $\frac{x}{2}$ by attaching a semicircle at each end. Express the total area A in terms of perimeter P.

10. _____



11. Find all real values of x for which $|4 - 6x| \leq 1$.

11. _____

12. Calculate the area of the region that is bounded by the circles $x^2 + y^2 = 1 \; and \; x^2 + y^2 - 8x - 9 = 0$.

12. _____

13. Find all real values of x which: $x^2 - 5x - 6 = 0$.

13. _____

14. How much water should be added to 3 liters of pure acid to yield a solution which is 15% acid?

14. _____

15. A line which passes through the point (0,5) is tangent to a circle of radius 1 centered at the origin at a point in the first quadrant. Find the coordinates of the point of tangency.

15. x = _____
y = _____

16. Find all real values of y for which $\sqrt{y+4} + \sqrt{9-y} = 5$.

16. _____

17. Find a formula for the inverse of the function f(x) = 3x – 4.

17. _____

18. If log represents $\log_4$, evaluate

$\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \ldots + \log \frac{98}{99} + \log \frac{99}{100}$

18. _____

19. A card is drawn at random from a well-shuffled deck of 52 cards containing 13 distinct cards in each of the four suits: hearts, diamonds, clubs, and spades. What is the probability of drawing neither a spade nor club?

19. _____

20. Given the parametric equations $x = t^3 + 9 \; and \; y = \frac{3}{4}t^3 + 7$ represent a line, what is the y-intercept of that line?

20. _____

1990 Junior Examination
(Section II)

1. Simplify: $\frac{3^2\ 2^3}{6^6}$ 1. _____

2. If f(x) = $\frac{1}{x}$ and g(x) = $\frac{1}{x+3}$, what is the domain $(f \circ g)(x)$? 2. _____

3. Simplify the following product to a single rational expression in lowest terms: 3. _____

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{29^2}\right)$$

4. Perform the indicated operations and simplify: 4. _____

$$\frac{\frac{x-y}{x+y}+\frac{x+y}{x-y}}{\frac{xy}{x-y}}$$

5. Find the minimum value of the function g(x) = $3x^2 - 5x + 2$ 5. _____

6. Write as a single fraction and simplify using only positive exponents: 6. _____

$(x+\ 1)^{\frac{-3}{4}} - (x+1)^{\frac{1}{4}}$

7. Find all real values of x for which $|3x + 1| = |x - 2|$. 7. _____

8. Quadrilateral ABCD is inscribed in circle P. DAB measures 95°, what is the measure of of angle DCB? 8. _____

9. Find the coordinate of the vertex of the Parabola $y^2 + 2y - 8x - 3 = 0$. 9. x= _____
y= _____

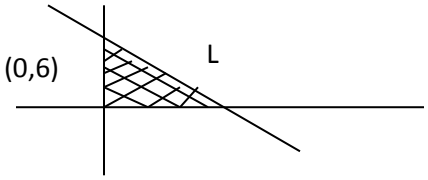
10. At a point on the ground 20 ft. from a building A surveyor observes the angle of indication to the top of the building to be $\frac{\pi}{3}$ radius. How tall is the building? 10. _____

11. List all solutions to $\sin\theta + \cos\theta = 1$ satisfying $0 \leq \theta < 2\pi$. 11. _____

12. What is the range of the function $g(x) = \frac{x-4}{x-5}$ 12. _____

13. Find all real values of x for which $(x^2 + x + 4)^{3/4} = 8$ 13. _____

14. The y-intercept of the line L in the figure below is 6. If the shaded area is 72 square units, what is the slope of L? 14. _____



15. List the letters of all the following relations which are NOT functions. 15. _____

a.) $\{(1,2),(3,2),(4,5)\}$ b.) $\{(x,y)|y = 3x + 7\}$

c.) $\{(x,y)|x^2 + y^2 = 3\}$ d.) $\{(x,y)|2x^2 + 3y = 4\}$

16. Find all real values of x for which $\frac{x+2}{x-3} < 0.$ 16. _____

17. Find the point(s) of intersection of the line $2x + y = 6$ and the parabola $y = 2x^2 - 7x + 6$ 17. _____

18. If $x + y = 15$ what is the minimum value of the quantity $x^2 + y^2$? 18. _____

19. Sales in a store were \$45,000 in 1980 and \$75,000 in 1986. Assuming a linear trend, estimate the 1989 sales figure. 19. _____

20. A 50lb. mixture of fertilizers x, y, and z containing 19% phosphate, 34% potash, and 47% nitrogen is desired. What amounts of the fertilizers x, y, and z are required if x, y, and z contain the following amounts of these chemical: 20. x= _____ lbs.___
y= _____ lbs.___
z= _____lbs.___

	Phosphate	Potash	Nitrogen
x	10%	30%	60%
y	20%	40%	40%
z	20%	30%	50%

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simply all fractions and radicals, and rationalize denominators. Your answers must be complete to receive credit for a problem.

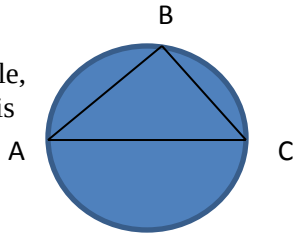
1. Suppose $f(x) = 4x - 3$. Find $f(a + 3)$. 1. _____

2. Simplify and express in terms of positive exponents: 2. _____
$$\frac{12r^{-6}s^{0}t^{-3}}{3r^{-4}s^{-3}t^{-5}}$$

3. Find all real values of x such that $|x + 3| = |x - 4|$ 3. _____

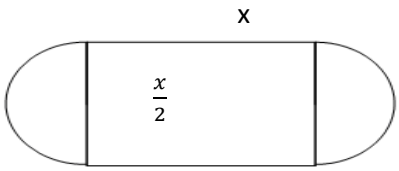
4. If sin x = 0.6 and $0 < x < \frac{\pi}{2}$, find sin 2x. 4. _____

5. Given that AC is the diameter of the circle, and that angle BAC measures 30°, what is the measure of angle BCA? 5. _____



6. Determine the x-intercepts of the circle $x^2 + y^2 - 8y - 16 = 0$. 6. _____

7. The figure below is constructed from a rectangle of the dimensions x and $\frac{x}{2}$ by attaching a semicircle at each end. Express the total area A in terms of perimeter P. 7. _____



8. Determine K so that the point (5,K) lies on the line joining (1,4) and (-3,1) 8. _____

9. Find all real values of x such that $10^{1-x} < 100$. 9. _____

10. Determine the rectangular coordinates (x,y) of a point whose polar coordinates are $\left(2, \frac{5\pi}{6}\right)$. 10. _____

11. Find the center and the radius of the circle Which passes through the points (7,1) , (5,5) and (-2,-2). 11. Center:_____ Radius:_____

12. Suppose the domain of f is the closed interval $[a, b]$, and suppose g is defined by g(x) = f (c-3x) for a constant c. what is the domain of g? 12. _____

13. Let $f(x) = \frac{x}{x-1}$. Find $f(f(x))$. 13. _____

14. Find all x in the interval $[0,2\pi)$ such that $|\cos x| \leq \frac{\sqrt{3}}{2}$. 14. _____

15. A line which passes through the point (0,5) is tangent to a circle of radius 1 centered at the origin at a point in the first quadrant. Find the coordinates of the point of tangency. 15. x = _____ y = _____

16. Determine k so that x + 2 is a factor of $x^3 + 3x^2 + kx - 2$. 16. _____

17. The area of a rectangle is 150 sq. ft., and the length of one side is three times the length of the adjacent side, find the dimensions of the rectangle. 17. _____

18. A card is drawn at random from a well shuffled deck of 52 cards containing 13 distinct cards in each of the four suits: hearts, diamonds, clubs, and spades. What is the probability of drawing a heart or a king? 18. _____

19. A multiple choice test consists of 5 questions, each of which has three possible answers. If a student guesses an answer to every question, how many possible ways are there for her to fill in the answer sheet? 19. _____

20. Find all real values x such that $\log x = \log(x + 3) - 1$. 20. _____

1990 Senior Examination
(Section II)

1. Express the quadratic $4x^2 - 4x + 2$ in the form $a(x-h)^2 + k$. 1. _____

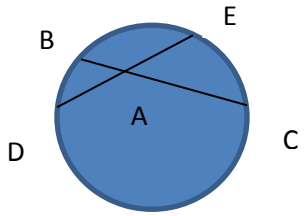
2. Find all real roots of the polynomial$x^3 + x^2 - x - 3$. 2. _____

3. Find the equation in slope-intercept form of the line which Passes through (2,-1) and is parallel to the line $y = 3x - 1$. 3. _____

4. Determine $\log_{16} 4$. 4. _____

5. Express in the form $a + bi: i\ (3-2i)^2$. 5. _____

6. Given AB = 2, AC = 12, and AE = 6, find DE. 6. _____



7. Find all real values of x for which the function $f(x) = \sqrt{x^2 - 2}$ is defined. 7. _____

8. Suppose that z is directly proportional to the sum of the squares of x and y, and that z = 50 when x = 4 and y = 1. Determine the constant of proportionality. 8. _____

9. Find the coordinates of the vertex of the parabola $y^2 + 2y - 8x - 3 = 0$. 9. _____

10. Simplify: $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$ 10. _____

11. Determine A, B, C such that

$$\frac{2x^2-21x+37}{(x-3)(x+1)(x-2)} = \frac{A}{x-3}+\frac{B}{x+1}+\frac{C}{x-2}$$

11. A= _____

B=_____

C= _____

12. Find the point(s) where the parabola $x^2 - 2x + y = 0$ intersects the line $x + 2y = 2$.

12. _____

13. List all vertical and horizontal asymptotes of the graph of $y = \frac{2x^2-x-3}{x^3-x}$.

13. _____

14. The Supreme Court has 9 members. In how many different ways can a 5-to-4 decision in favor of a judgment be reached?

14. _____

15. Two opposite vertices of a square are (a,a) and (4a,5a). Find the coordinates of the remaining two vertices.

15. _____

16. Quadrilateral ABCD is inscribed in circle P. If angle DAB measures 95°, what is the measure of angle DCB?

16. _____

17. Find all x in [0,2π) such that $\sin x = -\frac{1}{2}$.

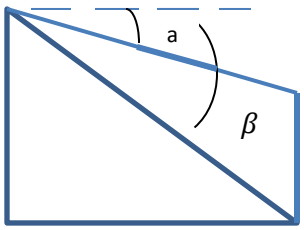
17. _____

18. Simplify: $\frac{1+\frac{1}{a+3}}{1-\frac{1}{a+3}}$.

18. _____

19. From the top of a taller building, the triangle a of depression to the top of a shorter building 30°, and the angle of β of depression to the base of the shorter building is 60°. Find the height of the taller building if the shorter building is 100 ft. tall.

19. _____



20. Isosceles triangle ABC has AB = AC = 13 and BC = 10. Find the length of the altitude to side AC.

20. _____