LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University - - 1991 Junior Examination
(Section 1)

NAME:__________________________________ **SCHOOL:**____________________________

Directions: For each problem, write your answer in the space provided.
Do not use approximations. Simplify all fractions and radicals.
Your answer must be complete to receive credit for a problem.

1. Find all real x for which: $3x^2 + 13x + 4 = 0$ 1. ________________

2. Find all coordinates of the point P, given that the point $(2,-1)$ bisects the line segment joining P and the point (4,3). 2. ________________

3. Find the area of the region in the xy plane which satisfies both of the following inequalities:

$$x^2 + y^2 + 6x - 2y \geq -9$$
$$x^2 + y^2 + 6x - 2y \leq -1$$

3. ________________

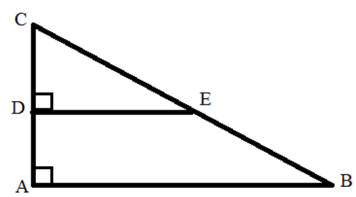
4. Find all real x for which $(x^2 - 4)b^x < 0$ if $b > 1$. 4. ________________

5. Express as a rational number, the repeating decimal

76.63424242 5. ________________

6. Find k if $x - 1$ is a factor of $k^2x^4 - 2kx^2 + 1$. 6. ________________

7. Find the area of right triangle DEC shown below if $\overline{DC} = 2$, $\overline{AC} = 3$, and $\overline{AB} = 10$. 7. ________________



8. In how many ways can the letters of the word “spectrum” be arranges so that the “r” and the “t” are always next to each other? 8. ________________

9. Find $\cos \dfrac{21\pi}{4}$. 9. ________________

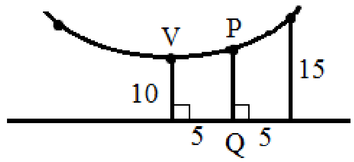
10. Find all real x for which: 10. ________________

$$\frac{2}{x+3} \geq \frac{1}{x-1}$$

11. Find the point(s) of intersection of the parabola $4y^2 + 4y - 5x + 12 = 0$ and the line $x = 9$. 11. ________________

(OVER)

12. If the curve shown is part of a parabola, with vertex V, find the distance from P to Q. 12. ____________



13. A function f, f, is called "even" if $f(x) = f(-x)$ for all x in its domain, or "odd" if $f(-x) = -f(x)$ for all x in its domain. If g is even, and h is odd and never zero, and both are defined for all reals, list all of the following that are odd:

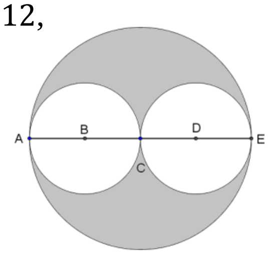
(a) $g(x)\,h(x)$ (c) $[h(x)]^2$ (e) $h(h(x))$

(b) $\frac{g(x)}{h(x)}$ (d) $g(x) + h(x)$ (f) $h(x) + h(-x)$

13. ____________

14. Solve for x: $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} = -2$ 14. ____________

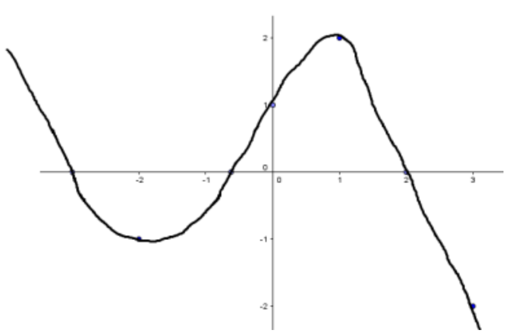
15. Find the area of the shaded region below if $\overline{AE} = 12$, $\overline{AC}$ is a diameter of the circle with center B, $\overline{CE}$ is a diameter of circle with center D, and $\overline{AE}$ is a diameter of circle with center C. 15. ____________



16. The perimeter of rectangle is 42 inches and the area is 108 sq. inches. Find the dimensions of the rectangle. 16. ____________

17. Find two consecutive positives integers whose product is 272. 17. ____________

18. List all of the statements (a) – (e) that are FALSE about the function f, whose graph is shown. 18. ____________



a) f has 5 roots

b) $f(0) - f(2) > 0$

c) $f(3) - 3 > 0$

d) $(f \circ f)(0) = 2$

e) f(x) is positive for all x in the interval (1,3]

19. If $\log x = \log_{10} x$, solve for x: $\log(x + 2) - \log x = 1$. 19. ____________

20. Three people play a game. At the end of each game, the one loser must double the money of each of the other two players. After three games, each has lost once, and each ends up with \$24. With how much did each person start? 20. ____________

1991 JUNIOR EXAMINATION
(Section II)

NAME:________________________________ SCHOOL:______________________________

1. Find all real x for which: $1 - \frac{3}{x} = \frac{40}{x^2}$ 1. ____________

2. Find the slope and y-intercept of the line whose equation is $3(x-2) + y = 7 - 6(y+4)$ 2. slope__________ y-intercept_______

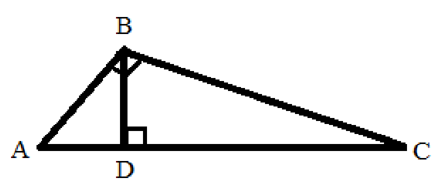
3. Find $\log_{1/2} 8$. 3. ____________

4. Find all real values of t for which the following function is defined: $f(t) = \sqrt{9 - (t-9)^2}$ 4. ____________

5. Find all real x for which: $\sqrt{x+3} = 2 + \sqrt{x-5}$ 5. ____________

6. Find the center and radius of the circle that passes through the points (0,5), (2,5) and (2,-1) 6. center __________ radius _________

7. In the right triangle ABC shown, find the length of $\overline{AC}$ if $\overline{AD} = 9$ and $\overline{BD} = 15$. 7. ____________



8. Find the distance between the parallel lines 12x – 5y = 2 and 12x – 5y = 7 8. ____________

9. A person has 8 shirts and 6 pairs of pants. How many different shirt-pants combinations can the person wear? 9. ____________

10. List all of the following expressions that are factors of $x^3 + 4x^2 + x - 6$:

a) $x - 1$ d) $x + 3$

b) $x^2 + 3x + 6$ e) $x^2 + 2x - 3$

c) $x - 2$

10. ____________

(OVER)

11. A belt just fits around three wheels with equations

$(x+10)^2 + y^2 = 1, (x-10)^2 + y^2 = 1,$

and $x^2 + (y - 10\sqrt{3})^2 = 1$.
Find the length of the belt.

11. ____________________

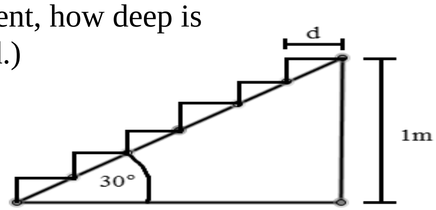
12. A regular hexagon (6 equal sides) is inscribed in a circle of radius 4. Find the area of this hexagon.

12. ____________________

13. If $0 \le t \le 2\pi$, $\sin t = 5/13$ and $\cos t = -12/13$, find $\cot t$.

13. ____________________

14. In the diagram of the stairs drawn to the right, if all the steps are congruent, how deep is each step? (That is, find d.)



14. __________________m

15. Find a formula for f(x) if f is a rational function whose graph passes through the point (2,5) and has only the asymptotes $y = 2x + 3$ and $x = 3$

15. ____________________

16. A three-digit number equals 19 times the sum of its digits. If the digits are reversed, the resulting number is greater than the given number by 297. The tens digit exceeds the units digit by 3. Find the given number.

16. ____________________

17. Determine the formula for f(x), if for all real Numbers a and b: $f(a)\,f(b) - f(ab) = a + b$.

17. ____________________

18. If $b > 1$ is any real number, find all values of x for which

$(\log_b x)^2 + 10 < 7 \log_b x$. (Your answer will be in terms of b.)

18. ____________________

19. Find the maximum and minimum values of the expression

$\cos t + \sin t$ when $0 \le t < 2\pi$.

19. ____________________

20. Find two points on the graph of $y = 3x$ where the distance to the origin is 2.

20. ____________________

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - - 1991 Senior Examination

(Section 1)

NAME:____________________________________ **SCHOOL:**______________________________

Directions: For each problem, write your answer in the space provided.
Do not use approximations. Simplify all fractions and radicals.
Your answer must be complete to receive credit for a problem.

1. Find all real x for which: $x^2 + 16x - 3 = 0$ 1. __________________

2. Find A, B, C, so that

$$\frac{6x^2 - 21x + 13}{(x^2+4)(x-5)} = \frac{Ax+B}{x^2+4} + \frac{C}{x-5}$$

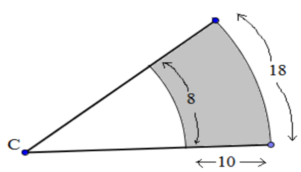
2. A = ______________
B = ______________
C = ______________

3. Assuming y is a differentiable function of x, find the derivative $\frac{dx}{dy}$: $x^2y + 5x = y^5 - 3$ 3. __________________

4. Find all real x for which the following statement is true: $|x| \leq x$ 4. __________________

5. Find all real x for which: $\sqrt{x+3} = 2 + \sqrt{x-5}$ 5. __________________

6. Find the shaded area of the polar rectangle shown. The two curves are arcs of concentric circles with center C. 6. __________________



7. Determine the larges and smallest values of $f(x) = 3x^4 + 4x^3 - 12x^2 + 1$ on the interval [-1,1].

7. largest_____________
smallest___________

8. List each of the following functions which is its own inverse: 8. __________________

a) f(x) = x c) h(x) = - x + 5 e) r(x) = 7x

b) g(x) = x + 5 d) k(x) = $\frac{1}{x}$

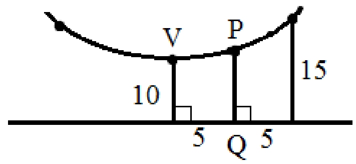
9. Find the domain of $(f \circ g)(x)$ when $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x-1}$. 9. __________________

10. If $f(x) = x^2 - x + 1$, what is the slope of the line joining the points (1,f(1)) and (2,f(2))? 10. __________________

(OVER)

11. Find k so that $g(x) = 9x^2 - 30x + k$ has exactly one real root. 11. ________________

12. If the curve shown is part of a parabola, with vertex V, find the distance from P to Q. 12. ________________



13. A function f, f, is called "even" if $f(x) = f(-x)$ for all x in its domain, or "odd" if $f(x) = -f(x)$ for all x in its domain. If g is even, and h is odd and never zero, both defined for all reals, and h(x) is never zero, list all of the following that are odd:

(a) $g(x)\,h(x)$ (c) $[h(x)]^2$ (e) $h(h(x))$ 13. ________________

(b) $\frac{g(x)}{h(x)}$ (d) $g(x) + h(x)$ (f) $h(x) + h(-x)$

14. Solve for x: $5^{x^2-x} = 25$ 14. ________________

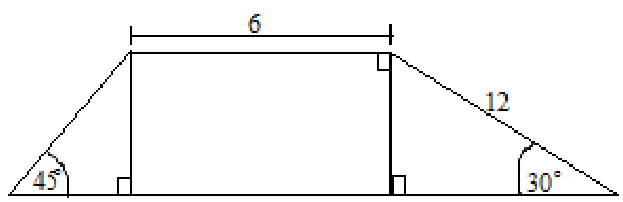
15. List all values of x where the function f is **not** differentiable: 15. ________________

$$f(x) = \begin{cases} -2x & \text{if } x < -1 \\ x^2 - 3 & \text{if } -1 \le x < 3 \\ 6x - 12 & \text{if } x \ge 3 \end{cases}$$

16. If $\log_b N = 5$, find $\log_{1/b} N$. 16. ________________

17. In how many ways can the letters of the word "spectrum" be arranges so that the "r" and the "t" are always next to each other? 17. ________________

18. Find the perimeter of the polygon drawn: 18. ________________



19. Find: $\sin 1° + \sin 2° + \sin 3° + \cdots + \sin 359°$ 19. ________________

20. Three people play a game. At the end of each game, the one loser must double the money of each of the other two players. After three games, each has lost once, and each ends up with $24. With how much did each person start? 20. ________________

1991 SENIOR EXAMINATION
(Section II)

NAME:________________________________ SCHOOL:______________________________

1. If g(x) = 4x, list **all** of the following that are true for all real x:

   a) $g(x^2) = (g(x))^2$ c) $g(-x) = g(x)$

   b) $g(|x|) = |g(x)|$ d) $g(3x) = 3g(x)$

1. ________________

2. ________________

2. If b > 1 is any real number, find all values of x for which $(\log_b x)^2 + 10 < 7 \log_b x$. (Your answer will be in terms of b.)

3. Determine a and b so that the following function is everywhere continuous:

$$f(x) = \begin{cases} 1 - x & \text{if} \quad x < 0 \\ ax + b & \text{if} \quad 0 \le x < 3 \\ 2 & \text{if} \quad x \ge 3 \end{cases}$$

3. ________________

4. A number is called "perfect" if it is the sum of all its positive integral divisors except itself. The number 6 is perfect. Another perfect number is:

   a) 36 b) 24 c) 16 d) 28 e) 12

4. ________________

5. Determine the formula for f(x), if for all real numbers a and b: f(a)f(b) - f(ab) = a + b.

5. ________________

6. center ____________
   radius ___________

6. Find the center and radius of the circle that passes through the points (0,5), (2,5) and (2,-1)

7. If θ is a fourth quadrant angle whose terminal side coincides with the line 3x + 4y = 0, find sin θ.

7. ________________

8. Find the distance between the parallel lines 12x – 5y = 2 and 12x – 5y = 7.

8. ________________

9. Find all real values of x for which $|x^2 - 5x - 5| > 9$.

9. ________________

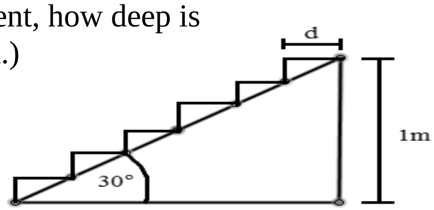
10. If the sum of an infinite geometric series if $S = \frac{a}{1-r}$, where a if the first term of the series and r is the common ration between successive terms, find the sum of the series:

$$2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \cdots$$

10. ________________

(OVER)

11. In the diagram of the stairs drawn to the right, if all the steps are congruent, how deep is each step? (That is, find d.)



11. ______________ m

12. Find: $\lim_{x \to \frac{\pi}{3}} \frac{\sin^3 x - \sin^3 \frac{\pi}{3}}{x - \frac{\pi}{3}}$

12. ________________

13. Suppose f and g are everywhere differentiable. Use the following table to determine $(g \circ f)'(3)$:

x	f(x)	f'(x)	g(x)	g'(x)
3	5	2	-1	4
5	7	3	6	8

13. ________________

14. If $y = \sin(\frac{1}{t})$, where does the graph of y cross the t axis on the interval $0 < t \leq -1$ if t is measured in radians?

14. ________________

15. Write the Cartesian equation (in the form y = mx + b) corresponding to the parametric equations: x = 3 – 2t, y = 4 + 3t.

15. ________________

16. A three-digit number equals 19 times the sum of its digits. If the digits are reversed, the resulting number is greater than the given number by 297. The tens digit exceeds the units digit by 3. Find the given number.

16. ________________

17. A belt just fits around three wheels with equations

$(x+10)^2 + y^2 = 1$, $(x-10)^2 + y^2 = 1$, and $x^2 + (y - 10\sqrt{3})^2 = 1$. Find the length of the belt.

17. ________________

18. Write in simplest form: $\frac{\sec\theta \csc\theta}{\tan\theta + \cot\theta}$

18. ________________

19. Find the largest possible domain for $f(x) = \frac{\ln(x+2)}{x-4}$

19. ________________

20. Find two points on the graph of y = 3x where the distance to the origin is 2.

20. ________________