

**LUZERNE COUNTY MATHEMATICS CONTEST**

Luzerne County Council of Teachers of Mathematics

Wilkes University - - 1992 Junior Examination

(Section I)

NAME: \_\_\_\_\_

SCHOOL: \_\_\_\_\_

Address: \_\_\_\_\_

Tel. no.: \_\_\_\_\_

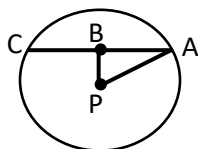
Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1. One number is twice another. The sum of their reciprocals is 2. Find the numbers. 1. \_\_\_\_\_

2. Suppose that b is inversely proportional to the square of a, and that b = 18 when a = 3. Find b when a is 9. 2. \_\_\_\_\_

3. Determine t so that the line through (-1,1) and (3,2) is parallel to the line through (0,6) and (-8,t). 3. \_\_\_\_\_

4. The given circle is centered at P, and  $\overline{PB} \perp \overline{AC}$ . If AC = 56 and AP = 35, find PB.



4. \_\_\_\_\_

5. Solve for x:  $\frac{1}{x-1} + \frac{1}{x+1} = \frac{3x}{x^2 - 1}$  5. \_\_\_\_\_

6. Determine all values of  $\theta$  in the interval  $[0, 2\pi]$  which satisfy  $\cos \theta = -\frac{1}{2}$  and  $\csc \theta = \frac{2}{\sqrt{3}}$ . 6. \_\_\_\_\_

7. Evaluate the following limit:  $\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$  7. \_\_\_\_\_

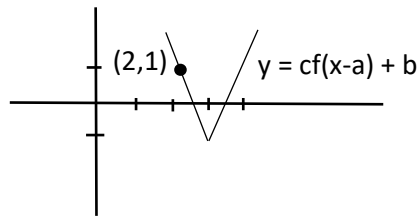
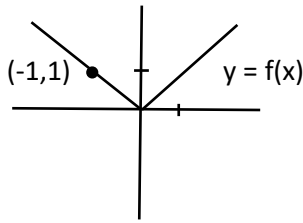
8. An executive committee consists of four women and six men. Three members will be selected at random to attend a conference in Hawaii. What is the probability that three women will be selected? 8. \_\_\_\_\_

9. A triangle has vertices (1,0), (5,0), and (3,5). What is the area of the triangle? 9. \_\_\_\_\_

10. Determine b so that  $\log_b 8 = \frac{3}{4}$ . 10. \_\_\_\_\_

**(OVER)**

11. By comparing the graphs below, determine the values of the constants  $a$ ,  $b$ , and  $c$ . (Each tick mark represents one unit.)



11. \_\_\_\_\_

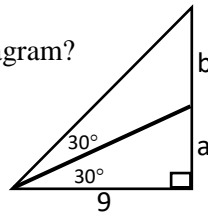
12. Suppose  $f(x) = \frac{2x^2 - 5x - 3}{x^2 - 3x}$ . Give the equations of the vertical and horizontal asymptotes of the graph of  $f$ .

12. \_\_\_\_\_

13. Find all values of  $x$  which satisfy  $x^2 + x < 2$ .

13. \_\_\_\_\_

14. What is the value of  $b$  in the given diagram?



15. Suppose Holly drove 90 miles at an average speed of 30 mph. What average speed would be necessary for her return trip in order to achieve an average speed of 50 mph for her entire trip?

15. \_\_\_\_\_ mph

16. Suppose  $f(x) = 2x^2 - x + 1$  and  $g(x) = 5x + 3$ . Find  $f(g(x))$  and simplify.

16. \_\_\_\_\_

17. Determine  $x$  such that  $\frac{1}{4^{x-2}} = 64$ .

17. \_\_\_\_\_

18. A man invests \$2200 in 3 accounts that pay 6%, 8%, and 9%, in annual interest, respectively. He has three times as much invested at 9% as he does at 6%. If his total interest for one year is \$178, how much money is invested at each rate?

18. \$ \_\_\_\_\_ (6%)  
 \$ \_\_\_\_\_ (8%)  
 \$ \_\_\_\_\_ (9%)

19. A woman bought some plates;  $\frac{2}{3}$  of them were cracked,  $\frac{1}{2}$  of them were chipped, and  $\frac{1}{4}$  were both chipped and cracked. Only 2 of the plates were neither chipped nor cracked. How many plates did she buy?

19. \_\_\_\_\_

20. If  $r > 0$  and  $(r + r^{-1})^2 = 5$ , find the value of  $r^3 + r^{-3}$ .

20. \_\_\_\_\_

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1. Determine the slope of the line with equation  $5x + 6y = 3$ . 1. \_\_\_\_\_

2. Suppose that  $b$  is directly proportional to  $a$ , and that  $b$  is 10 when  $a$  is 2. Find  $b$  when  $a$  is 6. 2. \_\_\_\_\_

3. A light-year is approximately  $5.9 \times 10^{12}$  miles. If a certain galaxy is  $1.7 \times 10^6$  light-year from our galaxy, find the distance in miles between the two galaxies. 3. \_\_\_\_\_ mi

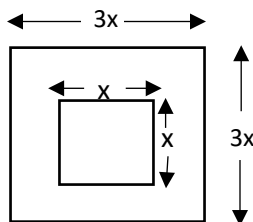
4. Find the  $x$ -intercepts of the graph of the function  $f(x) = 4x^2 + 4x - 3$ . 4. \_\_\_\_\_

5. Suppose a certain triangle is isosceles, with base 10 and perimeter 36. What is the area of the triangle? 5. \_\_\_\_\_

6. Solve for  $a$ :  $\left| \frac{3}{5}a + \frac{1}{2} \right| = 1$ . 6. \_\_\_\_\_

7. Determine all values of  $x$  in the interval  $[-2\pi, 0]$  which satisfy  $\tan x = \sqrt{3}$ . 7. \_\_\_\_\_

8. The figure on the right shows a dart board. If a dart hits the board at random, what is the probability that the dart will land in the center square?



8. \_\_\_\_\_

9. If  $\log a = x$  and  $\log b = y$ , express  $\log \frac{a^2}{b}$  in terms of  $x$  and  $y$ . 9. \_\_\_\_\_

10. A boat traveling at a constant speed takes 2 hours to travel 24 miles downstream and 3 hours to travel 18 miles upstream. What is the speed of the river current? 10. \_\_\_\_\_ mi/hr

(OVER)

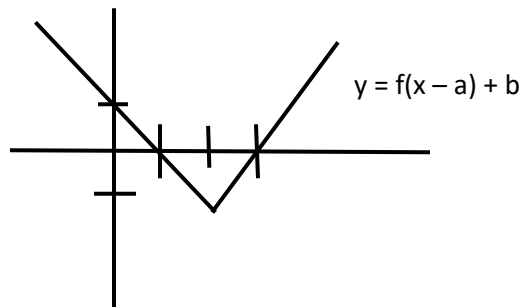
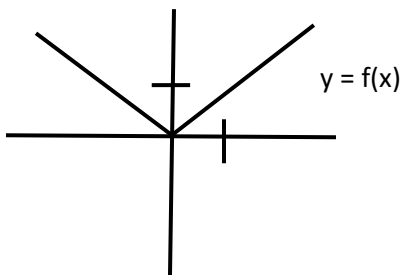
11. Determine the length of a chord that is a distance 5 from the center of a circle with radius 8. 11. \_\_\_\_\_  
\_\_\_\_\_

12. A motorcycle is traveling on a curve along a highway. The curve is an arc of a circle with radius  $\frac{1}{4}$  mile. If the motorcycle's speed is 42 miles per hour, what is the angle (in radians) through which the motorcycle will turn in  $\frac{1}{2}$  minute? 12. \_\_\_\_\_  
\_\_\_\_\_

13. The odds in favor of a certain team winning the World Series are 7:2. What is the probability that this team will win the World Series? 13. \_\_\_\_\_  
\_\_\_\_\_

14. Suppose a circle has center  $(-4,1)$  and a diameter with  $(2,6)$  as one endpoint. Find the coordinates of the other endpoint of the diameter. 14. \_\_\_\_\_

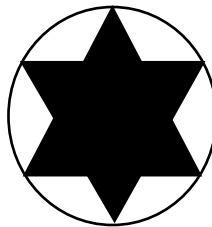
15. By comparing the graph below, determine the values of the constants  $a$  and  $b$ . (Assume that each tick mark represents one unit.) 15.  $a =$  \_\_\_\_\_  
 $b =$  \_\_\_\_\_



16. Suppose a line  $l$  contains a diameter of a circle  $2x^2 - 3x + 2y^2 + 5y - 2 = 0$ . If that diameter passes through the point  $(2,0)$ , find the slope-intercept form of the equation of  $l$ . 16. \_\_\_\_\_

17. Determine  $x$  such that  $\left(\frac{9}{25}\right)^x = \frac{5}{3}$ . 17. \_\_\_\_\_

18. Find the area of a regular 6-pointed star inscribed in a circle of radius 1. 18. \_\_\_\_\_



19. Suppose the trigonometric point  $P(t)$  on the unit circle has coordinate  $\left(-\frac{3}{5}, \frac{4}{5}\right)$ . Find the coordinates of  $P(2t)$ . 19. \_\_\_\_\_

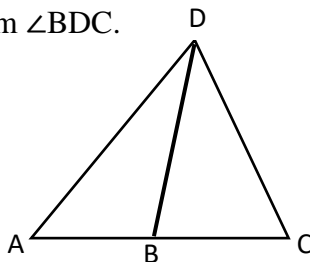
20. If  $r > 0$  and  $(r + r^{-1})^2 = 5$ , find the value of  $r^3 + r^{-3}$ . 20. \_\_\_\_\_

1992 JUNIOR EXAMINATION  
(SECTION II)

NAME: \_\_\_\_\_

SCHOOL: \_\_\_\_\_

1. A rectangle is 2 cm longer than it is wide. The diagonal of the rectangle is 10 cm long. Find the perimeter of the rectangle. 1. \_\_\_\_\_ cm
  
2. Find the equation of the circle with the center at the origin and y-intercepts 3 and -3. 2. \_\_\_\_\_
  
3. Determine all values of x which satisfy  $1 - 2x < 5$ . 3. \_\_\_\_\_
  
4. Determine the period of the function  $f(x) = 2\sin(6x - \pi)$ . 4. \_\_\_\_\_
  
5. If  $f(x) = \frac{2x-1}{x+3}$ , find  $f\left(\frac{1}{t}\right)$  and simplify. 5. \_\_\_\_\_
  
6. Solve for x:  $\log_{10}(x - 2) = 2$ . 6. \_\_\_\_\_
  
7. In the given triangle,  $m \angle ADB = m \angle BDC$ .  
If  $AD = 21$ ,  $DC = 14$ ,  
and  $AC = 25$ , what is  $AB$ ? 7. \_\_\_\_\_



(OVER)

11. A woman has 23 coins with a total value of \$1.90.  
If the coins consist only of dimes and nickels, how many of each type does she have? 11. \_\_\_\_\_ dimes  
\_\_\_\_\_ nickles
12. At the moment when the angle of elevation of the sun is  $60^\circ$ ,  
a building's horizontal shadow is 50 meters long. How tall  
is the building? 12. \_\_\_\_\_ m
13. A company has determined that can sell  $x$  videotapes per  
day at the price of  $p$  dollars per tape, where  $x + 20p = 230$ .  
Express the revenue  $R$  as a function of  $p$ . 13.  $R(p) =$  \_\_\_\_\_
14. A teacher has 5 books, she wishes to arrange 3 of them  
on a shelf. In how many different ways can this be done? 14. \_\_\_\_\_
15. If  $\sin x = \frac{1}{5}$  and  $\frac{\pi}{2} < x < \pi$ , determine  $\cos x$ . 15. \_\_\_\_\_
16. Among 200 students at a certain school, 85 take physics,  
95 take chemistry, and 50 take both physics and chemistry.  
How many students take neither physics nor chemistry? 16. \_\_\_\_\_
17. Suppose a transformation of the plane maps each point  $(x,y)$   
to the point  $(5x - 3, 2y + 7)$ . Determine which point is  
mapped to the point  $(2,3)$ . 17. \_\_\_\_\_
18. Solve for  $x$  in terms of  $y$ :  $\frac{2x}{x+3} = y$  18. \_\_\_\_\_
19. Find all values of  $x$  in the interval  $[0,2\pi]$  which satisfy  
 $\sqrt{2} \sin x - 1 = 0$ . 19. \_\_\_\_\_
20. Determine the area of the region between the graphs of  
 $x^2 + y^2 = 1$  and  $|x| + |y| = 1$ . 20. \_\_\_\_\_

1992 JUNIOR EXAMINATION  
(SECTION II)

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1. A circle has center (2,5) and passes through the point (-1,4).  
What is the radius of the circle? 1. \_\_\_\_\_

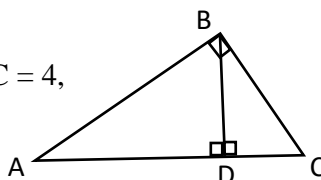
2. Determine a and b so that  $x^2 - 10x + 23 = (x - a)^2 + b$ . 2. a = \_\_\_\_\_  
b = \_\_\_\_\_

3. Find all values of x which satisfy  $|3x - 5| = 7$ . 3. \_\_\_\_\_

4. Determine the coordinates of the intersection point of the  
graphs of  $y = 1 - x^2$  and  $y = 2x + 2$ . 4. \_\_\_\_\_

5. What is the coefficient of  $x^4y^6$  in the expansion of  $(x + y^2)^7$ ? 5. \_\_\_\_\_

6. In the triangle ABC, if  $AD = 9$  and  $DC = 4$ ,  
what is  $BD$ ?



6. \_\_\_\_\_

7. Suppose an operation  $*$  is defined by  $x * y = 1 + xy$ .  
Which statement below is true? 7. \_\_\_\_\_

- (a)  $*$  is associate but not commutative
- (b)  $*$  is commutative but not associative
- (c)  $*$  is both associative and commutative
- (d)  $*$  is neither associative nor commutative

8. Determine the period of the function  $f(x) = 2\sin(6x - \pi)$ . 8. \_\_\_\_\_

9. A company has determined that it can sell  $x$  videotapes per day  
at the price of  $p$  dollars per tape, where  $x + 20p = 230$ .  
Express the revenue  $R$  as a function of  $p$ . 9.  $R(p) =$  \_\_\_\_\_

10. Suppose  $f$  is defined by  $f(x) = \frac{3x-1}{2x+5}$ . Give a formula  
for the inverse function  $f^{-1}$ . 10. \_\_\_\_\_

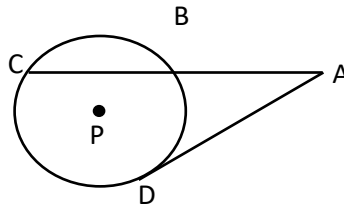
(OVER)

11. Determine a so that the following function g is continuous at 2:

$$g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ ax + 6 & \text{if } x \geq 2 \end{cases}$$

11. \_\_\_\_\_

12. In the given figure,  $\overline{AD}$  is tangent to the circle centered at P, and  $\overline{AC}$  is a secant. If  $m\widehat{BD} = 30^\circ$ ,  $m\widehat{DC} = 140^\circ$ , find  $m\angle BAD$ .



12. \_\_\_\_\_

13. Suppose a transformation of the plane maps a point  $(x,y)$  to the point  $(2x + y, x - 3y)$ . Determine which point is mapped to the point  $(1,2)$ .

13. \_\_\_\_\_

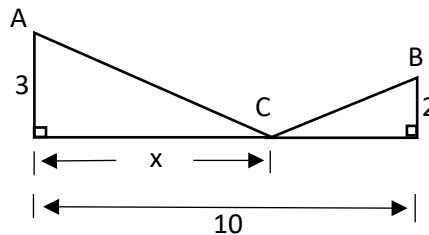
14. Eric has 3 math books and 2 science books; he wishes to arrange all of them on a shelf. How many arrangements are possible if books of the same type must be grouped together?

14. \_\_\_\_\_

15. A bag contains 5 red and 3 green marbles. Two marbles are selected at random, without replacement. What is the probability that the first marble is red and the second is green?

15. \_\_\_\_\_

16. In the given figure, determine x so that the path ACB has minimum length.



16. \_\_\_\_\_

17. Find all values of x in the interval  $[0,2\pi]$  which satisfy  $\sqrt{2} \sin x - 1 = 0$ .

17. \_\_\_\_\_

18. Solve for x:  $\log_{49} x = -\frac{1}{2}$ .

18. \_\_\_\_\_

19. Determine the value of the following sum:

$$\sin^2 1^\circ + \sin^2 2^\circ + \dots + \sin^2 88^\circ + \sin^2 89^\circ$$

19. \_\_\_\_\_

20. Find the area of the region between the graphs of  $x^2 + y^2 = 1$  and  $|x| + |y| = 1$ .

20. \_\_\_\_\_