

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University – 2001 Senior Examination

(Section I)

Name: _____ Address: _____

School: _____ City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1.) Find the equation of the line in slope-intercept form, that is

perpendicular to the line with the equation $y = -\frac{1}{3}x + \frac{18}{11}$ that

passes through the point (2, 7).

1) _____

2) The angles of a pentagon are in arithmetic progression.

One of these angles, in radians, must be

A) $\frac{\pi}{2}$ B) $\frac{3\pi}{5}$ C) $\frac{2\pi}{5}$ D) $\frac{3\pi}{10}$

E) none of the above

2) _____

3) Find the equation of the parabola with vertex (2, -4) and

the line $x = 2$ as its axis of symmetry that passes through the point (3, -1).

3) _____

4) Given $f(x) = 3x^4 + 2$ and assuming $h \neq 0$, evaluate the expression $[f(x+h) - f(x)]/h$

4) _____

5) Convert the repeating decimal $0.\overline{281} = 0.281281\dots$ into a fraction expressed in lowest terms.

5) _____

6) A 300-seat theater charges \$6 per adult ticket and \$3 per child ticket. If $\frac{2}{3}$ of the seats were filled and the ticket sales totaled \$921, how many *adults* were in the audience?

6) _____

7) Suppose $f(x) = \frac{x^2}{x-1}$ and $g(x) = \sqrt{x-5}$. What is the domain of $(f \circ g)(x)$?

7) _____

8) Find the value of $\cos(2\text{Arctan}\frac{1}{5})$.

8) _____

9) List all roots of multiplicity 2 for the polynomial

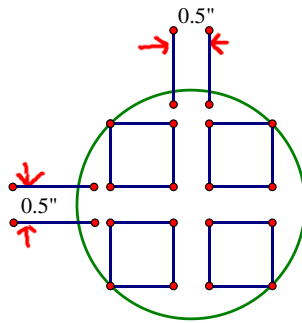
$$p(x) = x^4 - 2x^3 + x^2$$

9) _____

10) A particle projected vertically upward reaches an elevation of $h(t) = 160t - 16t^2$ feet at the end of t seconds. What is

the maximum height the particle can reach?

- 11) How many real values of x satisfy the following equation?
- 12) What is the minimum circumference of a circular pan needed to hold 4 slices of french toast, each 4.25 inches by 4.25 inches, with 0.5 inches between them?



10) _____ feet

11) _____

12) _____ inches

- 13) Assume that x is a real number such that $\cos x \neq 0$. the quantity $3\sin^2 x + 2\sec^2 x + 3\cos^2 x - 2\tan^2 x$ is equal to
- A) 5 B) $4 + \cos^2 x + \sin^2 x$ C) $4 + \csc^2 x - \cot^2 x$
- D) both A) and B) E) both A) and C)
- E) all of the above

13) _____

14) Evaluate the limit $\lim_{x \rightarrow -\infty} \frac{9+5x}{\sqrt{4x+7x^2-3}}$.

14) _____

15) Completely factor the expression $x^3 - 7x^2 + 14x - 8$.

15) _____

16) Find a number k such that $x + 4$ is a factor of

$$x^3 + kx^2 + 11x + 2k$$

16) _____

17) Find the constant term in the expansion of $\left(y + \frac{1}{2y}\right)^8$

17) _____

18) What is the degree measure of each interior angle of a regular octagon?

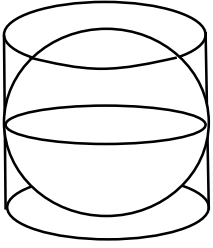
18) _____ degrees

19) Find all the real numbers x satisfying the inequality

$$x^3 \leq 3x - 2x^2$$

19) _____

20) A sphere is inscribed within a closed right circular cylinder whose height is twice its diameter. Express the surface area of the cylinder to that of the sphere as a fraction.



20) _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) A metal strip has a density of 250 gm/cm. Express the density in kg/m.

1) _____ kg/m

2) If a circular cake with a diameter of 8 in. costs \$6, what is a fair price for a circular cake with a diameter of 12 in.?

2) _____

3) If $f(x) = 3x^2 + x - 5$ and $h \neq 0$, evaluate the expression

$$\frac{f(x+h)-f(x)}{h}$$

3) _____

4) Determine all real numbers a such that $f(x) = x^3 - x^2 + ax - a$ has only real roots.

4) _____

5) What is the probability of rolling a sum of 9 on a pair of fair, six-sided dice?

5) _____

6) What is the units digit of 2137^{753}

6) _____

7) The arithmetic mean of a set of 20 numbers is 80. If two members of the set, namely 52 and 72, are discarded, what is the arithmetic mean of the remaining set of numbers?

7) _____

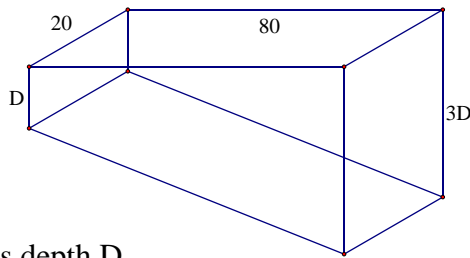
8) Find the exact value of $\sin \frac{7\pi}{12}$.

8) _____

9) What is the period of $f(x) = 4 \cos \left(-2x + \frac{\pi}{4} \right)$?

9) _____

10) A 20 ft by 80 ft rectangular swimming pool is constructed so that its maximum depth is 3 times its minimum depth D .



Write an expression for the

volume of the pool in terms of its depth D .

10) _____ ft^3

11) Find all values of x such that the distance between $(-7, 3)$ and $(x, 5)$ is 6.

11) _____

12) Determine the domain of the function $\frac{\sqrt{x+1}\sqrt{x-5}}{x+2}$

12) _____

13) Evaluate the $\lim_{x \rightarrow 0} \frac{x^3 - x}{x^2 + x}$.

13) _____

14) Find the vertex and axis of symmetry for the parabola that satisfies the equation $y = 3x^2 - 6x + 22$.

14) _____

15) Solve the following inequality:

$$|x + 2| > |x - 4|$$

15) _____

16) Suppose $f(x) = x^2 + 2$. Find and completely expand $(f \circ f \circ f)(x)$.

16) _____

17) Find all real numbers x satisfying the equation

$$\ln(2x - 3) - \ln(x + 4) = \frac{1}{2}$$

17) _____

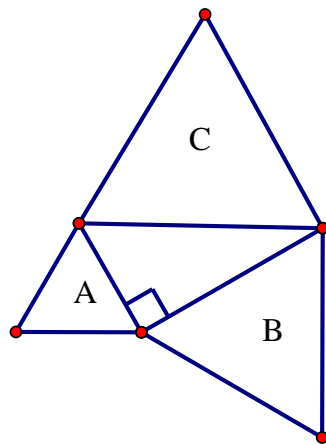
18) In how many ways can a baker sell eight identical apple pies to four different customers so each customer gets at least one pie?

18) _____

19) What is the volume of a right circular cone with height 5 in. and base radius 4 in.?

19) _____ in³

20) In the following figure, assume A, B, and C are equilateral triangles. A has an area of $9\sqrt{3}$ square units and B has an area of $16\sqrt{3}$ units. Find the area of C.



20) _____ sq. units

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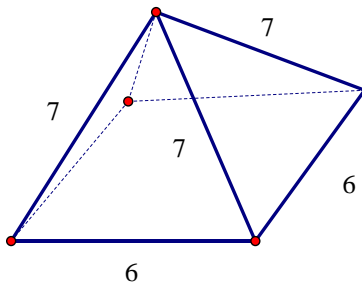
Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the circumference of a circle with diameter 8 units. 1) _____ units

2) Find the equation of the line in slope-intercept form that passes through the points (1, 3) and (-6, 8). 2) _____

3) How many distinct 6-letter strings (words) can be generated from the word ALASKA? 3) _____

4) Find the total surface area of the following square pyramid.



4) _____ sq. units

5) If $\ln x = A$ and $\ln y = B$, then write the following as a single term involving A and B . 5) _____

6) Given an equilateral triangle with sides of length L , express the ratio of the area of the triangle to its perimeter in terms of L . 6) _____

7) The graph of $x^2 - 9y^2 = 0$ is
A) a parabola B) an ellipse C) a point
D) two distinct straight lines E) none of the above
7) _____

8) Find the center and radius of the circle that satisfies the equation $x^2 + y^2 - 6x + 12y - 55 = 0$
8) center: _____
radius: _____ units

9) Solve for a : $\sqrt{8+a} = \sqrt{4+a} + 2$ 9) $a =$ _____

10) Find all real numbers x and y that satisfy the following equations simultaneously:

$$\frac{27^{x+y}}{3^{2y}} = 9, \quad \frac{8^{x+y}}{16^x} = 64$$

10) $x =$ _____

$y =$ _____

11) What is the probability of rolling a regular six-sided die exactly 3 times without rolling a “4” before rolling a “4” on the fourth roll?

11) _____

12) The radius of a sphere is 3 inches. What change in the radius will increase the volume by $207\pi \text{ in}^3$?

12) _____ inches

13) Find the equation of the tangent line in slope- intercept form to the circle $(x + 2)^2 + (y - 1)^2 = 20$ through $(-4, 5)$.

13) _____

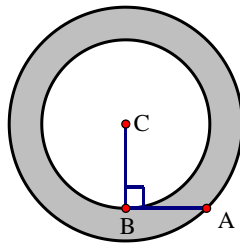
14) What is sum of the numeric coefficients in the complete expansion of $(x^2 - 2xy + y^2)^7$?

14) _____

15) Express the complex number $-\sqrt{3} - i$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 \leq \theta \leq 2\pi$.

15) _____

16) In the figure below, C is the center of both circles $CB = 7$ units and $AB = 9$ units. Find the area of the shaded region.



16) _____ sq. units

17) Rationalize the denominator and simplify:

$$\frac{5}{\sqrt[3]{23}}$$

17) _____

18) If $x > 0, y > 0$, express the following in the form $\frac{x^a}{y^b}$, where $a, b \geq 0$.

$$\frac{\left(\sqrt[5]{x^{-12}y^{\frac{18}{7}}}\right)^{-5/6}}{\sqrt[4]{y^3}}$$

18) _____

19) List all roots of multiplicity 2 for the polynomial

$$p(x) = x^3 - 5x^2 + 8x - 4$$

19) _____

20) A merchant has 6 pounds of mixed nuts that cost \$15.

He wants to add peanuts that cost \$1.50 per pound and cashews that cost \$5 per pound to obtain 40 pounds of a mixture the costs \$3.50 per pound. How many pounds of peanuts and cashews are needed?

20) _____ pounds

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) If a circular cake with a diameter of 8 in. costs \$6, what is a fair price for a circular cake with diameter 12 in.?

1) _____

2) What is the probability of rolling a sum of 6 on a pair of fair, six-sided dice?

2) _____

3) Write as a single fraction:

$$\frac{\frac{2+5}{3+7}}{|5-\sqrt{37}|} \cdot \frac{\frac{1}{3} \div \frac{3}{4}}{\sqrt{37}+5}$$

3) _____

4) A line passes through the points $(-3, 8)$ and $(A, 4)$, and has an x -intercept at $x = 2$. Find A .

4) _____

5) Find all real numbers x which satisfy $\frac{x}{x-3} > -4$

5) _____

6) Suppose $M = 12 \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right)$ and

$$N = \frac{7}{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right).$$

Express MN in the form $C + Di$, where C, D are real numbers.

6) _____

7) Find the real number x which maximizes the function

$$f(x) = 8x - 3x^2.$$

7) _____

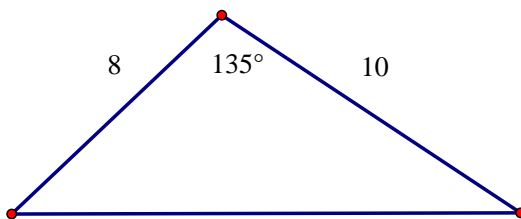
8) Find the 37th term of the arithmetic progression whose first three terms are 2, 5, and 8, respectively

8) _____

9) Suppose $\cos \theta = \frac{2}{5}$, where $\frac{3\pi}{2} < \theta < 2\pi$. Find the exact value of $\tan 2\theta$.

9) _____

10) Find the area of the following triangle:



10) _____ sq. units

- 11) The three numbers 4, 5, and 8 are:
- a) the side lengths of an acute triangle.
 - b) the side lengths of an obtuse triangle.
 - c) the side lengths of a right triangle.
 - d) not the side lengths of any triangle.

11) _____

- 12) A farmer bought 749 sheep. She sold 700 of them for the price paid for the original 749 sheep. The remaining 49 sheep were sold at the same price per sheep at which the other 700 sheep were sold. Based on the cost, what is the percent gain on the entire transaction?

12) _____

- 13) Find the equation of the line in slope-intercept form that passes through the point $(-1, 2)$ and is perpendicular to the line $y = x + \frac{x-3}{2} - \frac{1}{4}$.

13) _____

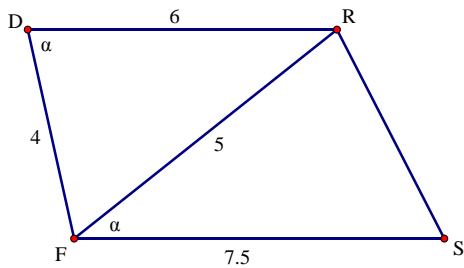
- 14) Assume $B = 10, \frac{G}{8} = R, 5Y = B$ and $2R = Y$.
Find $(3B + R)(Y/G)$

14) _____

- 15) Suppose that f is a function such that $f(4) = f(5) = 2$
 $f(6) = 1$ and $f(n + 1) = \frac{f(n-2)f(n)+4}{f(n-1)}$

15) _____

- 16) In the following figure, how long is \overline{RS} ?



16) _____ units

- 17) Find all real numbers x which satisfy $\frac{e^x - 5e^{-x}}{4} = 1$.

17) _____

- 18) Given real numbers x, y, z such that $0 < x \leq 2y - z$, which of the following statements are true?
- a) $z \leq 2y - x$
 - b) $2y \geq z + x$
 - c) $0 < (2y - z)/x$
 - d) $-x \geq z - 2y$
 - e) All of the above statements are true

18) _____

- 19) Find real numbers A and B such that

$$\frac{7x+5}{x^2+x-2} = \frac{A}{x+2} + \frac{B}{x-1}$$

19) _____

20) A point P is taken interior to a square of side length a . P is equidistant from two consecutive vertices of the square and from the side opposite these consecutive vertices. If d is the common distance, express d in terms of a .

20) _____