

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2011 Junior Examination

(Section I)

NAME: _____

Address: _____

SCHOOL: _____

City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Suppose the perimeter of a square is increased by 8 units. If the area of the new square is 196 square units, what is the length of a side of the original square? 1) _____

2) There are 3 math courses and 4 science courses offered in a school. If a student wants to select 3 courses with at least one course from the math courses and one course from the science courses, how many choices does he/she have? 2) _____
(a) 30 (b) 35 (c) 42 (d) 48

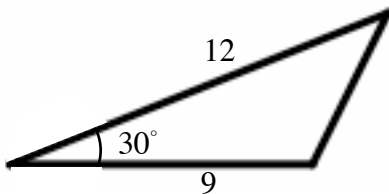
3) Express $10^{3 \log 5} + \log_4 16^{20}$ as an integer. 3) _____

4) Find the distance between $P = (2, -3)$ and $Q = (6, 4)$. 4) _____

5) An instructor is writing a *true* or *false* quiz with 10 questions and wants 4 questions to have *true* as the answer. How many different versions of the quiz are possible? 5) _____

6) Find all vertical asymptotes of the function $f(x) = \frac{x - 3}{x^2 - x - 6}$. 6) $x =$ _____

7) What is the area of the triangle shown below? 7) _____



8) Find all real solutions to the equation $(x - 5)(x - 6) = x - 5$. 8) $x =$ _____

9) The sum of the squares of three consecutive even integers is 980. Find the three integers. 9) _____

10) How many rational roots does $f(x) = x^3 - 5x^2 - 2x + 24$ have? 10) _____
(a) 0 (b) 1 (c) 2 (d) 3

(OVER)

11) What is the area of the region determined by $\begin{cases} x \geq 0 \\ x - y - 1 \geq 0 \\ 3x - 2y - 6 \leq 0 \end{cases}$? 11) _____

12) What is the domain of the function $f(x) = \sqrt{\frac{x+3}{x^2-1}}$? 12) _____

13) Find the period of $y = 5\cos(4x + 3\pi)$. 13) _____

14) Express $\frac{(5+i)(4-i)}{2i-3}$ in the form $a + bi$. 14) _____

15) How many solutions does $\tan\frac{x}{2} - \cos x = 0$ have on $[0, 2\pi]$?
 (a) 0 (b) 1 (c) 2 (d) 3 15) _____

16) Let $f(x) = \begin{cases} \log_2 x, & x > 0 \\ \log_{\frac{1}{2}} |x|, & x < 0 \end{cases}$. Then $f(a) > f(-a)$ for which values of a ? 16) _____

- (a) $(-1, 0) \cup (0, 1)$ (b) $(-\infty, -1) \cup (1, +\infty)$
 (c) $(-1, 0) \cup (1, \infty)$ (d) $(-\infty, -1) \cup (0, 1)$

17) If $\cos\left(\frac{5\pi}{12} + \alpha\right) = \frac{1}{3}$ and $-\pi < \alpha < -\frac{\pi}{2}$, then $\cos\left(\frac{\pi}{12} - \alpha\right) =$ _____. 17) _____

18) Let $f(x) = x^2 - |x|$. What values of m satisfy $f(-m^2 - 1) < f(2)$? 18) _____

19) If $a = \sqrt{7} - 1$, then $3a^3 + 12a^2 - 6a - 12 =$ _____. 19) _____

20) If a circle on the left side of the y -axis has a center on the x -axis and a radius of $\sqrt{5}$, and is tangent to the straight line $x + 2y = 0$, then the equation of the circle is 20) _____
 (a) $(x - \sqrt{5})^2 + y^2 = 5$
 (b) $(x + \sqrt{5})^2 + y^2 = 5$
 (c) $(x - 5)^2 + y^2 = 5$
 (d) $(x + 5)^2 + y^2 = 5$

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(Section II)

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) If the area of an equilateral triangle is $7\sqrt{3}$ square units, what is the length of a side of the triangle? 1) _____
- 2) What is the circumference of a circle that is centered at (2, 6) and contains the point (4, 9)? 2) _____
- 3) Find all real solutions to the inequality $|x + 3| - |x - 2| \geq 3$. 3) _____
- 4) Find all real solutions to $(9x^2)2^x - 2^x = 0$. 4) $x =$ _____
- 5) Factor $x^3 + 2x^2 + 4x + 8$ completely. 5) _____
- 6) Find the vertex of the parabola $2x^2 + 8x + 1$. 6) _____
- 7) $(\sqrt{3} + i)^8$ is equal to : 7) _____
(a) $-128 - 128\sqrt{3}i$ (b) $128\sqrt{3} - 128i$
(c) $-128\sqrt{3} + 128i$ (d) $-128 + 128\sqrt{3}i$
- 8) A rectangle is 3 times as long as it is wide. The perimeter of the rectangle is 32 inches. What is the width of the rectangle? 8) _____ inches
- 9) What is the exact value of $\cos\left(\arcsin\frac{3}{8}\right)$? 9) _____
- 10) Find all values B such that the slope of the line passing through the points (3, -4) and (7, B) equals $-\frac{1}{5}$. 10) $B =$ _____

(OVER)

LUZERNE COUNTY MATHEMATICS CONTEST

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(Section I)

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the least common multiple of 60 and 100? 1) _____

2) The graph of the equation $2x^2 + 3xy + 6y^2 - 4x - 5y - 7 = 0$ is a(n) 2) _____
(a) ellipse (b) circle (c) parabola (d) hyperbola

3) What is the last digit in the number 7^{338} ? 3) _____

4) Find all real solutions to the equation $e^{4x} + 4e^{2x} - 32 = 0$. 4) $x =$ _____

5) What is the horizontal asymptote of the function 5) $y =$ _____
 $f(x) = \frac{9x^2 + 6x + 1}{2x^2 + x + 6}$?

6) Find all real solutions to the inequality $\sqrt{2x^2 + 1} - x \leq 1$. 6) _____

7) $(\sin x + \cos x)^2 =$ _____. 7) _____
(a) $1 + \sin 2x$ (b) $1 + \cos 2x$ (c) $1 + 2\sin x \cos x$
(d) both a and c (e) both b and c

8) Suppose $f(x) = \lfloor \log x \rfloor$. If $a \neq b$ and $f(a) = f(b)$, then $a + b$ is 8) _____
in the interval
(a) $(1, \infty)$ (b) $[1, \infty)$ (c) $(2, \infty)$ (d) $[2, \infty)$

9) If $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\sin \alpha = \frac{3}{5}$, then $\tan\left(\alpha + \frac{\pi}{4}\right) =$ _____. 9) _____

10) Three students and two teachers stand in a line. How many different 10) _____
lines can be formed in which the two teachers are not next to
to each other?

(OVER)

11) Find the sum of $1 + 8 + 15 + 22 + \dots + 204$. 11) _____

12) If $f(x)$ satisfies $f(x + y) = f(x) + f(y) + 2xy$, $x, y \in \mathbb{R}$, and $f(1) = 2$, then $f(-2) =$ _____. 12) _____

13) Find all real solutions to $x^{\frac{11}{6}} + x^{\frac{5}{3}} - 2x^{\frac{3}{2}} = 0$ 13) $x =$ _____

14) How many terms in the expression $(x + \sqrt[4]{3} y)^{20}$ have rational coefficients? 14) _____

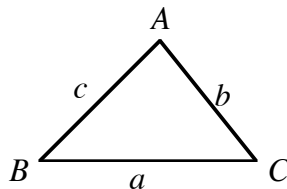
15) What is the solution to the inequality $\left| \frac{x-2}{x} \right| > \frac{x-2}{x}$? 15) _____
(a) $(0, 2)$ (b) $(-\infty, 0)$
(c) $(2, \infty)$ (d) $(-\infty, 0) \cup (0, \infty)$

16) Find all values of x such that $(k - 3)x + (4 - k)y + 1 = 0$ is parallel to $2(k - 3)x - 2y + 3 = 0$. 16) $k =$ _____

17) What is the remainder when $x^{2011} + 2011x^{2010} + x^2 + x + 1$ is divided by $x + 1$? 17) _____

18) In a triangle ABC , if $a^2 - b^2 = \sqrt{3} bc$, and $\sin C = 2\sqrt{3} \sin B$, then $A =$ _____. 18) _____

- (a) 30° (b) 60°
(c) 120° (d) 150°



19) Find the equation of the line tangent to the circle $x^2 + y^2 = 74$ at the point $(-5, 7)$. Write your answer in slope-intercept form. 19) _____

20) If x, y satisfy $\begin{cases} x + 2y \leq 4 \\ x - y \leq 1 \\ x + 2 \geq 0 \end{cases}$, what is the maximum value of z if $z = 3x - y$? 20) _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) How many of the statements below are always true? 1) _____
i) $\sqrt{x^2} = x$
ii) all squares are rectangles
iii) $f(x) = \frac{x^2 - 1}{x + 1}$ has a vertical asymptote at $x = -1$
(a) 3 (b) 2 (c) 1 (d) 0
- 2) Factor $x^3 - 2x^2 + x$ completely. 2) _____
- 3) If $3x^2 - 4x - 5 = 7$, then $x^2 - \frac{4}{3}x - 5 =$ _____. 3) _____
- 4) What is the area of the triangle with vertices $A(2, 4)$, $B(8, 8)$ and $C(-4, 13)$? 4) _____
- 5) If $M + T = H$, $A + M = T$, and $A = 2T$, what does $M + A + T + H$ equal in terms of T ? 5) _____
- 6) What is the negation of the statement: *It always rains on Thursday.* 6) _____
(a) It never rains on Thursday.
(b) There exists a Thursday on which it does not rain.
(c) If it is Thursday, it cannot be raining.
(d) None of the above.
- 7) A googol is 10^{100} and a googolplex is 10^{googol} , Find 7) _____
 $\frac{\log(\log(\text{googolplex}))}{4}$.
- 8) $f(x) = \begin{cases} 2^x + 1, & x < 1 \\ x^2 + ax, & x \geq 1 \end{cases}$. If $f(f(0)) = 4a$, then $a =$ _____. 8) $a =$ _____
- 9) A complex number z satisfies $(1 + 2i)z = 4 + 3i$. Express z in the form $a + bi$. 9) $z =$ _____
- 10) Find all real solutions to the inequality $\frac{1}{|x + 5|} \geq 4$ 10) _____

(OVER)

- 11) How many revolutions will a car wheel of diameter 32 in. make as the car travels a distance of $\frac{1}{2}$ mile (2640 feet)? 11) _____
- 12) Given $f(x) = x^2 + x$ and $h \neq 0$, compute and simplify $\frac{f(x+h) - f(x)}{h}$. 12) _____
- 13) The root of $f(x) = e^x + x - 2$ is in the interval
 (a) $(-2, -1)$ (b) $(-1, 0)$
 (c) $(0, 1)$ (d) $(1, 2)$ 13) _____
- 14) If the coefficient of x^3 in $\left(x + \frac{a}{x}\right)^5$ is 10, then $a =$ _____. 14) $a =$ _____
- 15) In a triangle ABC , D and E are on the sides \overline{AB} and \overline{AC} , respectively. $\overline{DE} \parallel \overline{BC}$. If $\frac{\overline{AD}}{\overline{AB}} = \frac{3}{4}$ and $\overline{AE} = 6$, then $\overline{AC} =$ _____. 15) $\overline{AC} =$ _____
- 16) A number is called *perfect* if it is the sum of all its positive integral divisors except itself. The number 6 is perfect. Another perfect number is
 (a) 36 (b) 24 (c) 18 (d) 28 16) _____
- 17) If an odd function $f(x)$ is increasing on $(0, \infty)$, and $f(1) = 0$, then $\frac{f(x) - f(-x)}{x} < 0$ for which values of x ? 17) _____
 (a) $(-1, 0) \cup (1, \infty)$ (b) $(-\infty, -1) \cup (0, 1)$
 (c) $(-\infty, -1) \cup (1, \infty)$ (d) $(-1, 0) \cup (0, 1)$
- 18) Let $A = \left\{ (x, y) \mid \frac{x^2}{4} + \frac{y^2}{16} = 1 \right\}$ and $B = \left\{ (x, y) \mid y = 3^x \right\}$. How many subsets does $A \cap B$ have? 18) _____
 (a) 4 (b) 3 (c) 2 (d) 1
- 19) Find all values of m such that the straight line $\sqrt{3}x - y + m = 0$ is tangent to the circle $x^2 + y^2 - 2x - 2 = 0$. 19) $m =$ _____
- 20) If x and y satisfy $\begin{cases} y \leq 1 \\ x + y \geq 0 \\ x - y - 2 \leq 0 \end{cases}$, what is the maximum value of z if $z = x - 2y$? 20) _____