LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2011 Junior Examination
(Section I)

NAME: ____________________ **Address:** ____________________

SCHOOL: ____________________ **City/ZIP:** ____________________

Telephone: ____________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Suppose the perimeter of a square is increased by 8 units. If the area of the new square is 196 square units, what is the length of a side of the original square?

1) 12

2) There are 3 math courses and 4 science courses offered in a school. If a student wants to select 3 courses with at least one course from the math courses and one course from the science courses, how many choices does he/she have?
(a) 30 **(b)** 35 **(c)** 42 **(d)** 48

2) *a*

3) Express $10^{3\log 5} + \log_4 16^{20}$ as an integer.

3) 165

4) Find the distance between $P = (2, -3)$ and $Q = (6, 4)$.

4) $\sqrt{65}$

5) An instructor is writing a *true* or *false* quiz with 10 questions and wants 4 questions to have *true* as the answer. How many different versions of the quiz are possible?

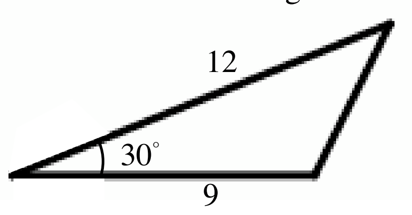
5) 210

6) Find all vertical asymptotes of the function $f(x) = \dfrac{x-3}{x^2 - x - 6}$.

6) $x =$ -2

7) What is the area of the triangle shown below?

7) 27



8) Find all real solutions to the equation $(x-5)(x-6) = x - 5$.

8) $x =$ 5, 7

9) The sum of the squares of three consecutive even integers is 980. Find the three integers.

9) -20,-18,-16 and 16,18,20

10) How many rational roots does $f(x) = x^3 - 5x^2 - 2x + 24$ have?
(a) 0 **(b)** 1 **(c)**2 **(d)** 3

10) *d*

(OVER)

11) What is the area of the region determined by $\begin{cases} x \ge 0 \\ x - y - 1 \ge 0 \\ 3x - 2y - 6 \le 0 \end{cases}$? **11)**______4______________

12) What is the domain of the function $f(x) = \sqrt{\dfrac{x+3}{x^2 - 1}}$? **12)**__$[-3,-1) \cup (1, \infty)$________

13) Find the period of $y = 5\cos(4x + 3\pi)$. **13)**______$\dfrac{\pi}{2}$______________

14) Express $\dfrac{(5+i)(4-i)}{2i - 3}$ in the form $a + bi$. **14)**______5 - 3i______________

15) How many solutions does $\tan\dfrac{x}{2} - \cos x = 0$ have on $[0, 2\pi]$? **15)**______b______________

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

16) Let $f(x) = \begin{cases} \log_2 x, \ x > 0 \\ \log_{\frac{1}{2}} |x|, \ x < 0 \end{cases}$. Then $f(a) > f(-a)$ for which values of a ? **16)**______c______________

(a) $(-1, 0) \cup (0, 1)$ **(b)** $(-\infty, -1) \cup (1, +\infty)$

(c) $(-1, 0) \cup (1, \infty)$ **(d)** $(-\infty, -1) \cup (0, 1)$

17) If $\cos\left(\dfrac{5\pi}{12} + \alpha\right) = \dfrac{1}{3}$ and $-\pi < \alpha < -\dfrac{\pi}{2}$, then $\cos\left(\dfrac{\pi}{12} - \alpha\right) =$ _______ . **17)**______$\dfrac{-2\sqrt{2}}{3}$__________

18) Let $f(x) = x^2 - |x|$. What values of m satisfy $f(-m^2 - 1) < f(2)$? **18)**___-1 < m < 1 or | m | < 1______

19) If $a = \sqrt{7} - 1$, then $3a^3 + 12a^2 - 6a - 12 =$ _____. **19)**________24______________

20) If a circle on the left side of the y-axis has a center on the x-axis and a radius of $\sqrt{5}$, and is tangent to the straight line $x + 2y = 0$, then the equation of the circle is **20)**______d______________

(a) $\left(x - \sqrt{5}\right)^2 + y^2 = 5$

(b) $\left(x + \sqrt{5}\right)^2 + y^2 = 5$

(c) $(x - 5)^2 + y^2 = 5$

(d) $(x + 5)^2 + y^2 = 5$

LUZERNE COUNTY MATHEMATICS CONTEST

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Wilkes University - 2011 Junior Examination
(Section II)

NAME: ______________________ **Address:** ______________________

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Telephone: ______________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) If the area of an equilateral triangle is $7\sqrt{3}$ square units, what is the length of a side of the triangle ?

1)______$2\sqrt{7}$ ______________

2) What is the circumference of a circle that is centered at $(2, 6)$ and contains the point $(4, 9)$?

2)______$2\sqrt{13}\ \pi$ __________

3) Find all real solutions to the inequality $|x+3| - |x-2| \geq 3$.

3)_____$x \geq 1$ or $[1, \infty)$ _______

4) Find all real solutions to $(9x^2)2^x - 2^x = 0$.

4)_$x =$___ $\pm\frac{1}{3}$ ______________

5) Factor $x^3 + 2x^2 + 4x + 8$ completely.

5)___$(x^2+4)(x+2)$_______

6) Find the vertex of the parabola $2x^2 + 8x + 1$.

6)_____(-2, -7)_______________

7) $\left(\sqrt{3} + i\right)^8$ is equal to :

(a) $-128 - 128\sqrt{3}\,i$ **(b)** $128\sqrt{3} - 128i$

(c) $-128\sqrt{3} + 128i$ **(d)** $-128 + 128\sqrt{3}\,i$

7)______ *a* __________________

8) A rectangle is 3 times as long as it is wide. The perimeter of the rectangle is 32 inches. What is the width of the rectangle?

8)___________4_____inches__

9) What is the exact value of $\cos\left(\arcsin\frac{3}{8}\right)$?

9)_______ $\frac{\sqrt{55}}{8}$ ___________

10) Find all values B such that the slope of the line passing through the points $(3, -4)$ and $(7, B)$ equals $-\frac{1}{5}$.

10)_$B =$___ $\frac{-24}{5}$ or -4.8 ______

(OVER)

11) Let $a = \log_3 2$, $b = \ln 2$, and $c = 5^{-\frac{1}{2}}$, then

(a) $a < b < c$ **(b)** $b < c < a$
(c) $c < a < b$ **(d)** $c < b < a$

11)______a__________________

12) List all values of A such that $Ax^2 + 7x + 3 = 0$ has exactly one real solution.

12)_$A =$__0, $\frac{49}{12}$____________

13) Suppose $f(x) = x^2 - 1$. Find all values of m such that if $x \in \left[\frac{3}{2}, \infty\right)$, then $f\left(\frac{x}{m}\right) - 4m^2 f(x) \leq f(x-1) + 4f(m)$.

13)_____$|m| \geq \frac{\sqrt{3}}{2}$_________

14) For which values of b does $y = x + b$ intersect $y = 3 - \sqrt{4x - x^2}$?

(a) $\left[-1, 1 + 2\sqrt{2}\right]$ **(b)** $\left[1 - 2\sqrt{2}, 2 + 2\sqrt{2}\right]$
(c) $\left[1 - 2\sqrt{2}, 3\right]$ **(d)** $\left[1 - \sqrt{2}, 3\right]$

14)_____c____________________

15) Find all real solutions to $2\ln x + \ln 2x = 4\ln 4 - 3\ln 2$.

15)_$x =$__ $\sqrt[3]{16}$ or $2^{\frac{4}{3}}$______

16) A number is randomly selected from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability of getting a number which is a multiple of 3?

(a) $\frac{1}{5}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{1}{2}$

16)____c_____________________

17) Find the rectangular coordinates for the point that has polar coordinates $\left(2, \frac{5\pi}{6}\right)$.

17)___$\left(-\sqrt{3}, 1\right)$______________

18) Suppose $f(x) = x^2 + 1$. Find $(f \circ f \circ f)(x)$.

18)_$x^4 + 4x^6 + 8x^4 + 8x^2 + 5$__

19) If $f(x) = 2\cos 2x + \sin^2 x - 4\cos x$, then the minimum value of $f(x)$ = ______ .

19)_$\frac{-7}{3}$______________________

20) Let $D = \left\{ (x, y) \;\middle|\; \begin{array}{l} x + y - 11 \geq 0 \\ 3x - y + 3 \geq 0 \\ 5x - 3y + 9 \leq 0 \end{array} \right\}$.

20)_____a____________________

If $y = a^x$ intersects the region D, then the value of a is completely determined by which interval ?

(a) $(1, 3]$ **(b)** $[2, 4)$ **(c)** $(1, 4]$ **(d)** $[3, \infty)$

LUZERNE COUNTY MATHEMATICS CONTEST

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the least common multiple of 60 and 100? **1)** 300

2) The graph of the equation $2x^2 + 3xy + 6y^2 - 4x - 5y - 7 = 0$ is a(n) **2)** *a*

(a) ellipse **(b)** circle **(c)** parabola **(d)** hyperbola

3) What is the last digit in the number 7^{338} ? **3)** 9

4) Find all real solutions to the equation $e^{4x} + 4e^{2x} - 32 = 0$. **4)** $x = \frac{\ln 4}{2}$ or $\ln 2$

5) What is the horizontal asymptote of the function **5)** $y = \frac{9}{2}$

$$f(x) = \frac{9x^2 + 6x + 1}{2x^2 + x + 6} \text{ ?}$$

6) Find all real solutions to the inequality $\sqrt{2x^2 + 1} - x \leq 1$. **6)** [0, 2]

7) $(\sin x + \cos x)^2 =$ _____ . **7)** *d*

(a) $1 + \sin 2x$ **(b)** $1 + \cos 2x$ **(c)** $1 + 2\sin x \cos x$
(d) both **a** and **c** **(e)** both **b** and **c**

8) Suppose $f(x) = |\log x|$. If $a \neq b$ and $f(a) = f(b)$, then $a + b$ is in the interval **8)** *c*

(a) $(1, \infty)$ **(b)** $[1, \infty)$ **(c)** $(2, \infty)$ **(d)** $[2, \infty)$

9) If $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\sin\alpha = \frac{3}{5}$, then $\tan\left(\alpha + \frac{\pi}{4}\right) =$ _____ . **9)** $\frac{1}{7}$

10) Three students and two teachers stand in a line. How many different lines can be formed in which the two teachers are not next to to each other? **10)** 72

(OVER)

11) Find the sum of $1 + 8 + 15 + 22 + \cdots + 204$.

11) 3075

12) If $f(x)$ satisfies $f(x + y) = f(x) + f(y) + 2xy, \quad x, y \in \mathbb{R}$, and $f(1) = 2$, then $f(-2) =$ _____ .

12) 2

13) Find all real solutions to $x^{\frac{11}{6}} + x^{\frac{5}{3}} - 2x^{\frac{3}{2}} = 0$

13) $x =$ 0, 1

14) How many terms in the expression $\left(x + \sqrt[4]{3}\, y\right)^{20}$ have rational coefficients?

14) 6

15) What is the solution to the inequality $\left|\dfrac{x - 2}{x}\right| > \dfrac{x - 2}{x}$?

(a) $(0, 2)$ **(b)** $(-\infty, 0)$

(c) $(2, \infty)$ **(d)** $(-\infty, 0) \cup (0, \infty)$

15) *a*

16) Find all values of *x* such that $(k - 3)x + (4 - k)y + 1 = 0$ is parallel to $2(k - 3)x - 2y + 3 = 0$.

16) $k =$ 3, 5

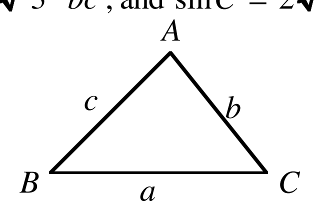
17) What is the remainder when $x^{2011} + 2011x^{2010} + x^2 + x + 1$ is divided by $x + 1$?

17) 2011

18) In a triangle *ABC*, if $a^2 - b^2 = \sqrt{3}\, bc$, and $\sin C = 2\sqrt{3}\sin B$, then $A =$ _____ .

(a) 30° **(b)** 60°

(c) 120° **(d)** 150°



18) *b*

19) Find the equation of the line tangent to the circle $x^2 + y^2 = 74$ at the point $(-5, 7)$. Write your answer in slope-intercept form.

19) $\dfrac{5}{7}x + \dfrac{74}{7}$

20) If x, y satisfy $\begin{cases} x + 2y \le 4 \\ x - y \le 1 \\ x + 2 \ge 0 \end{cases}$, what is the maximum value of z if $z = 3x - y$?

20) 5

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) How many of the statements below are always true? **1)**__________2_______________

i) $\sqrt{x^2} = x$

ii) all squares are rectangles

iii) $f(x) = \dfrac{x^2 - 1}{x + 1}$ has a vertical asymptote at $x = -1$

(a) 3 **(b)** 2 **(c)** 1 **(d)** 0

2) Factor $x^3 - 2x^2 + x$ completely. **2)**______$x(x-1)^2$___________

3) If $3x^2 - 4x - 5 = 7$, then $x^2 - \frac{4}{3}x - 5 =$ _____ . **3)**________-1________________

4) What is the area of the triangle with vertices $A(2, 4)$, $B(8, 8)$ and $C(-4, 13)$? **4)**______39__________________

5) If $M + T = H$, $A + M = T$, and $A = 2T$, what does $M + A + T + H$ equal in terms of T? **5)**________$2T$_________________

6) What is the negation of the statement: *It always rains on Thursday*. **6)**_______*b*_________________

(a) It never rains on Thursday.
(b) There exists a Thursday on which it does not rain.
(c) If it is Thursday, it cannot be raining.
(d) None of the above.

7) A *googol* is 10^{100} and a *googolplex* is 10^{googol}, Find $\dfrac{\log(\log(googolplex))}{4}$. **7)**________25_________________

8) $f(x) = \begin{cases} 2^x + 1, & x < 1 \\ x^2 + ax, & x \geq 1 \end{cases}$. If $f(f(0)) = 4a$, then $a =$ _____ . **8)**__$a =$ 2_________________

9) A complex number z satisfies $(1 + 2i)z = 4 + 3i$. Express z in the form $a + bi$. **9)**__$z =$ $z - i$__________________

10) Find all real solutions to the inequality $\dfrac{1}{|x + 5|} \geq 4$ **10)**_$\left[\frac{-21}{4}, -5\right) \cup \left(-5, \frac{-19}{4}\right]$____

(OVER)

11) How many revolutions will a car wheel of diameter 32 in. make as the car travels a distance of $\frac{1}{2}$ mile (2640 feet)?

11) $\frac{990}{\pi}$

12) Given $f(x) = x^2 + x$ and $h \neq 0$, compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

12) $2x + h + 1$

13) The root of $f(x) = e^x + x - 2$ is in the interval

(a) $(-2, -1)$ **(b)** $(-1, 0)$

(c) $(0, 1)$ **(d)** $(1, 2)$

13) *c*

14) If the coefficient of x^3 in $\left(x + \dfrac{a}{x}\right)^5$ is 10, then $a =$ _____ .

14) $a = 2$

15) In a triangle ABC, D and E are on the sides $\overline{AB}$ and $\overline{AC}$, respectively. $\overline{DE} \parallel \overline{BC}$. If $\dfrac{\overline{AD}}{\overline{AB}} = \dfrac{3}{4}$ and $\overline{AE} = 6$, then $\overline{AC} =$ _____ .

15) $\overline{AC} = 8$

16) A number is called *perfect* if it is the sum of all its positive integral divisors except itself. The number 6 is perfect. Another perfect number is

(a) 36 **(b)** 24 **(c)** 18 **(d)** 28

16) *d*

17) If an odd function $f(x)$ is increasing on $(0, \infty)$, and $f(1) = 0$, then $\dfrac{f(x) - f(-x)}{x} < 0$ for which values of x?

(a) $(-1, 0) \cup (1, \infty)$ **(b)** $(-\infty, -1) \cup (0, 1)$

(c) $(-\infty, -1) \cup (1, \infty)$ **(d)** $(-1, 0) \cup (0, 1)$

17) *d*

18) Let $A = \left\{(x, y) \;\middle|\; \dfrac{x^2}{4} + \dfrac{y^2}{16} = 1\right\}$ and $B = \left\{(x, y) \;\middle|\; y = 3^x\right\}$. How many subsets does $A \cap B$ have?

(a) 4 **(b)** 3 **(c)** 2 **(d)** 1

18) *a*

19) Find all values of m such that the straight line $\sqrt{3}\,x - y + m = 0$ is tangent to the circle $x^2 + y^2 - 2x - 2 = 0$.

19) $m = \sqrt{3}, -3\sqrt{3}$

20) If x and y satisfy $\begin{cases} y \leq 1 \\ x + y \geq 0 \\ x - y - 2 \leq 0 \end{cases}$, what is the maximum value of z if $z = x - 2y$?

20) 3