LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University - 2012 Junior Examination
(Section I)

NAME: _____   Address: _____

SCHOOL: _____   City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Factor the following completely : $3x^2 - 3y^2 + 18x + 18y$.

1)_____$3(x+y)(x-y-6)$_____

2) What is the area of an equilateral triangle that has a perimeter of 30 units?

2)_____$25\sqrt{3}$_____

3) Find the radius of the circle if an arc of length 8 m on the circle subtends a central angle of $\dfrac{\pi}{3}$ radians.

3)_____$\dfrac{25}{\pi}$_____ m

4) Express the following as an integer: $\log_3 45 - \log_3 5 + \ln e^6$.

4)_____8_____

5) What is the period of $f(x) = \dfrac{5}{6}\sin\left(\dfrac{7}{8}x - 2\pi\right)$?

5)_____$\dfrac{16\pi}{7}$_____

6) Which of the following functions is equivalent to $y = x$ for all x ?

   (a) $y = \left(\sqrt{x}\right)^2$ (b) $y = \sqrt[3]{x^3}$

   (c) $y = \sqrt{x^2}$ (d) $f(x) = \dfrac{x^2}{x}$

6)_____b_____

7) If $x = \sqrt{2}$, then $\dfrac{x+2}{x+1} - \dfrac{x^2 - 2x + 1}{x^2 - 1} = $?

7)_____$3\left(\sqrt{2} - 1\right)$_____

8) If $4y^2 - my + 25$ is a complete square, then m is
   (a) 10   (b) ± 10 (c) 40   (d) ± 20

8)_____d_____

9) Express $(2 - 2i)^6$ as an imaginary number.

9)_____$512i$_____

10) Find all the real solutions to the inequality
$$\dfrac{x^2 - 12x + 32}{x - 8} \geq 0$$

10)_____$[4, 8) \cup (8, \infty)$_____

(OVER)

11) If the equation $x^4 + Ax^2 + Bx + C = 0$ has roots -1, 1, and 2, find C.

11) ___ $C = 4$ ___

12) What is the degree measure of an angle formed by the minute and hour hands of a clock at 3:06 pm. ?

12) ___ $57°$ ___

13) How many ways can 30 cents in change be paid if only dimes, nickels, and pennies can be used?

13) ___ 15 ___

14) Two types of coffee (*A* and *B*) are added to a bin containing 7 lb. of coffee that sells for $4.50 / lb. Coffee *A* costs $5.00 / lb. and coffee *B* costs $3.50 / lb. How many pounds of coffee *A* were added if the final mixture contains 40 lb. of coffee worth $4.75 / lb. ?

14) ___ $28\frac{2}{3}$ ___ lbs

15) If the coefficient of x^3 in the expansion of $\left(x + \dfrac{a}{x} \right)^5$ is 10, then $a = $?

15) ___ 2 ___

16) Joe is 6 feet tall. At 3:00 pm. his shadow is 8 ft. 4 in. long. The shadow of the flagpole he is standing by is 600 in. long. How tall, in feet, is the flagpole ?

16) ___ 36 ___ ft ___

17) If $\sin\alpha = \dfrac{1}{3}$, $\tan\alpha < 0$, then $\cos\alpha = = $ _____ .

(a) $-\dfrac{1}{3}$ **(b)** $\dfrac{1}{3}$

(c) $-\dfrac{2\sqrt{2}}{3}$ **(d)** $\dfrac{2\sqrt{2}}{3}$

17) ___ c ___

18) If $f(x) = \begin{cases} x^2 - x & , \quad x \le 0 \\ 1 + 2\log x, & x > 0 \end{cases}$ and $f(x) = 2$, then $x = $ _____.

18) ___ $\sqrt{10}$ ___

19) If $\left| x - \sqrt{3} \right| + \left(y + \dfrac{\sqrt{3}}{3} \right)^2 = 0$, then $(x \cdot y)^{2005} = $ _____.

19) ___ -1 ___

20) The smallest value of $f(x) = \dfrac{x^2 + 5}{\sqrt{x^2 + 4}}$ is

(a) 2 **(b)** $\dfrac{17}{4}$

(c) $\dfrac{5}{2}$ **(d)** $\dfrac{5}{4}$

20) ___ c ___

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find all real values of x such that the slope between the points $(2, 7)$ and $(x, x + 4)$ equals -3.

1) $x = $ ____ $\dfrac{9}{4}$ ____

2) Find all values of α such that $(x + 3)^2 + \alpha = 0$ has a real solution.

2) ____ $\alpha \le 0$ ____

3) What is the remainder when $f(x) = 7x^{57} + 12x^3 - 6x^2 + 8$ is divided by $(x - 1)$?

3) ____ 21 ____

4) Find all real solutions to $\log_5 (7 - x) = 2$.

4) ____ -18 ____

5) If the square root of $2x + 1$ is 5, then what is the cube root of $5x + 4$?

5) ____ 4 ____

6) Find all real values of x such that $\dfrac{x^2 - 2x - 3}{|x| - 1} = 0$.

6) ____ 3 ____

7) Fizzy Soda runs a contest in which 1 in 4 bottle caps wins a prize. If Sam buys 2 bottles of Fizzy Soda, what is the probability that he wins a prize?

7) ____ $\dfrac{7}{16}$ ____

8) If $a, b \in \mathbb{R}$, $\dfrac{1 + 2i}{a + bi} = 1 + i$, then

(a) $a = \dfrac{3}{2}, b = \dfrac{1}{2}$ (b) $a = 3, b = 1$

(c) $a = \dfrac{1}{2}, b = \dfrac{3}{2}$ (d) $a = 1, b = 3$

8) ____ a ____

9) What is the volume of a right circular cylinder that has a diameter of 12 units and a height of 24 units?

9) ____ 864π ____

10) What is $f\left[f\left(\dfrac{1}{4}\right)\right]$ if $f(x) = \begin{cases} \log_2 x & , x > 0 \\ 3^x & , x \le 0 \end{cases}$?

10) ____ $\dfrac{1}{9}$ ____

(OVER)

11) For $A \neq 0$, $B \neq 0$, $A \neq B$, express x and y in terms of A and B

$$Ax + By = 0$$
$$A^2x + B^2y = 2$$

11) $\displaystyle x = \frac{-2}{AB - A^2}$

$\displaystyle y = \frac{-2}{AB - B^2}$

12) Find all real solutions to $\sqrt{\sqrt{x}} = 3$.

12) _____ 81 _____

13) The solution to the inequality $\left| \dfrac{x}{1-x} \right| > \dfrac{x}{1-x}$ is

(a) $\{x \mid 0 < x < 1\}$ **(b)** $\{x \mid x < 0 \text{ or } x > 1\}$

(c) $\{x \mid x > 0\}$ **(d)** $\{x \mid x < 1\}$

13) _____ b _____

14) If $A \neq 0$, find all real solutions to $x^2 - \dfrac{2}{3}Ax - \dfrac{7A^2}{12} = 0$ expressed in terms of A.

14) $\displaystyle -\frac{1}{2}A, \ \frac{7}{6}A$

15) What is the partial sum s_n of the sequence below?

$a_0 = 1, \ a_1 = 1 + 2, \ a_2 = 1 + 2 + 4, \ \ldots, \ a_n = 1 + 2 + 4 + \ldots + 2^{n-1}$

(a) $2^{n+1} - 2 - n$ **(b)** $2^n - n - 1$

(c) $2^{n+2} - n - 3$ **(d)** $2^{n+2} - n - 2$

15) _____ a _____

16) If $x < \dfrac{5}{4}$, then what is the largest value attained by

$y = 4x + \dfrac{1}{4x - 5} - 2$?

16) _____ 1 _____

17) What is the smallest value of $3^x + 9^y$ if $x + 2y - 2 = 0$?

17) _____ 6 _____

18) If $f(x) = \sin x + \cos x$, then $f\left(\dfrac{\pi}{12}\right)$ is

(a) $\dfrac{2\sqrt{3}}{3}$ **(b)** $\dfrac{\sqrt{3}}{2}$ **(c)** $\dfrac{\sqrt{6}}{2}$ **(d)** $\dfrac{\sqrt{2}}{2}$
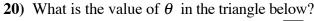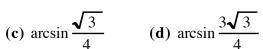
18) _____ c _____

19) If $z = x + 2y$ has the largest value of 3 in the region

$\begin{cases} x - y \leq 0 \\ x + y > 0 \quad ? \\ y \leq a \end{cases}$

then a equals _____.

19) _____ $a =$ 1 _____

20) What is the value of θ in the triangle below?

(a) $\arcsin \dfrac{3}{8}$ **(b)** $\arcsin \dfrac{3\sqrt{3}}{8}$

(c) $\arcsin \dfrac{\sqrt{3}}{4}$ **(d)** $\arcsin \dfrac{3\sqrt{3}}{4}$

20) _____ b _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the equation of the straight line, in slope-intercept form, passing through the point (-2, 3) and perpendicular to the straight line $2x + y - 5 = 0$?

1)_____$\dfrac{1}{2}x + 4$_____

2) Find all x such that the point $(x, 1)$ is a distance of 6 units from the point (3, -2).

2)___$x = 3 \pm 3\sqrt{3}$___

3) If $f(x) = \begin{cases} 3^x, & x \le 1 \\ -x, & x > 1 \end{cases}$ and $f(x) = 2$, then $x =$ _____.

3)___$x = \dfrac{\ln 2}{\ln 3}$___

4) Find $g\left(g\left(\dfrac{1}{2}\right)\right)$ if $g(x) = \begin{cases} e^x, & x \le 0 \\ \ln x, & x > 0 \end{cases}$.

4)_____$\dfrac{1}{2}$_____

5) If an 8 million dollar spending budget is first reduced by 20% and then raised by 30% the following year, what is the net increase in spending?

   (a) 10%   (b) 5%   (c) 4%   (d) no change

5)_____c_____

6) Rationalize the denominator of the expression $\dfrac{5x+1}{\sqrt[3]{x}}$.

6)_____$\dfrac{(5x+1)\sqrt[3]{x^2}}{x}$_____

7) Find the vertex of the parabola $y = 3x^2 + 10x - 28$.

7)___$\left(-\dfrac{5}{3}, -\dfrac{109}{3}\right)$___

8) There are 8 seats in the first row of the conference room. Three people will be seated. If there is an empty seat on each person's left and right, how many ways can the 3 people be seated?

   (a) 12   (b) 16   (c) 24   (d) 32

8)_____c_____

9) If $f_0(x) = \dfrac{1}{1+x}$ and $f_{n+1} = f_0 \circ f_n$ for $n = 0, 1, 2, \dots$, find $f_3(4)$.

9)_____$\dfrac{11}{17}$_____

10) If $\log_A x = 3$ and $\log_A y = 5$, what does $\log_A x^2 y^2$ equal ?

10)_____16_____

(OVER)

11) Find $f^{-1}(3)$ if $f(x + 1) = 2^x$.

11) $\dfrac{\ln 3}{\ln 2} + 1$

12) If $a, b \in \mathbb{R}$ and $(a + i)i = b + i$, then

    (a) $a = 1, b = 1$ (b) $a = -1, b = 1$

    (c) $a = -1, b = -1$ (d) $a = 1, b = -1$

12) _____ d _____

13) What is the value of $\dfrac{f(2)}{f(1)} + \dfrac{f(4)}{f(3)} + \dfrac{f(6)}{f(5)} + \ldots + \dfrac{f(2012)}{f(2011)}$

when $f(a + b) = f(a) \cdot f(b)$, $a, b \in \mathbb{R}$, and $f(1) = 2$.

13) _____ 2012 _____

14) What is the measure of each interior angle of a regular heptagon?

14) _____ $\dfrac{900°}{7}$ _____

15) The period of the function $g(x) = (\sin x + \cos x)^2$ is

    (a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) π (d) 2π

15) _____ c _____

16) What is the exact value of $\cos \dfrac{\pi}{8}$?

16) _____ $\dfrac{\sqrt{2 + \sqrt{2}}}{2}$ _____

17) If $(1 - 2x)^8 = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^8$, then $a_2 =$

    (a) 1792   (b) -1792   (c) 112   (d) -112

17) _____ c _____

18) In a sequence $\{a_n\}$, if $a_1 + 2a_2 + 3a_3 + \ldots + na_n = 2n$,
then $a_n =$ _____ .

18) $a_n = \dfrac{2}{n}$

19) Find all real solutions to the inequality $\dfrac{3}{|x + 6|} > 2$.

19) $(-\dfrac{15}{2}, -6) \cup (-6, -\dfrac{9}{2})$

20) If $a = \sqrt{3} - 1$, then $\left(\dfrac{a + 2}{1 - a^2} - \dfrac{2}{a + 1} \right) \div \dfrac{a}{a - 1} = $?

20) _____ $-\sqrt{3}$ _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations.
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1) Which of the statements below are true?

1)_____ d _____

   (a) Squares are not parallelograms.
   (b) Trapezoids are parallelograms.
   (c) Some rhombuses are not parallelograms.
   (d) Some rhombuses are squares.

2) If the length of an edge of a cube is doubled, the length
of the diagonal of the cube is increased by a factor of

2)_____ a _____

   (a) 2   **(b)** 8   **(c)** $2\sqrt{2}$ **(d)** $\sqrt{2}$

3) What is the domain of $f(x) = \sqrt{\dfrac{2x+1}{\pi - x}}$?

3)_____ $[-\dfrac{1}{2}, \pi)$ _____

4) $f(x)$ is an odd function and $f(x) = \sin x + x + b$,
then $b =$ _____ .

4) $b = $ 0 _____

5) Find $f(-8)$, when $f(x) = \begin{cases} 2^x & , x > 0 \\ f(x+3), x \leq 0 \end{cases}$

5)_____ 2 _____

6) If a set $M = \{ (x, y) \mid x + y = 1 \}$ and a set
$N = \{ (x, y) \mid x - y = 3 \}$, then $M \cap N =$

6)_____ d _____

   (a) $(-1, 2)$ **(b)** $(2, -1)$
   (c) $\{2, -1\}$ **(d)** $\{(2, -1)\}$

7) Find all real solutions to $4\log x = 2\log 2 + 2\log(3x - 4)$.

7) $x = $ $2, 4, 3 \pm \sqrt{17}$

8) The period of $f(x) = 8\sin(5x + 3\pi)$ is

8)_____ b _____

   (a) 8   **(b)** $\dfrac{2\pi}{5}$ **(c)** 3π **(d)** $\dfrac{5\pi}{2}$

9) What is the value of $x^2 - 4x - 6$ when $x = 2 - \sqrt{10}$?

9)_____ 0 _____

10) Find the sum of $2 + 6 + 10 + \ldots + 142$.

10)_____ 2592 _____

(OVER)

11) What is the equation of the line in slope-intercept form of the line perpendicular to $7x + 5y = 8$ and passing through the midpoint of the points (5, 7) and (9, -3) ?

11) $y = \dfrac{5}{7}x - 3$

12) The oldest child in a family if 1.5 times as old as the youngest. The two middle children are 13 and 15 years old. If the average age of all four children is $14\dfrac{1}{2}$ years, how old is the oldest child?

12)_____ 18 _____

13) Find $\tan\alpha$ if $\cos\alpha = -\dfrac{3}{5}$ and $\alpha \in \left(\pi, \dfrac{3\pi}{2}\right)$,

13)_____ $\dfrac{4}{3}$ _____

14) How many solutions does $\cos 3x \sec x = 2\cos 3x$ possess in the interval $[0, 2\pi]$?

14)_____ c _____

   (a) 2   **(b)** 4   **(c)** 6   **(d)** 8

15) A pizza parlor offers a plain cheese pizza and a choice of 11 toppings. How many different types of pizza can be ordered?

15)_____ 2048 _____

16) What is the sum of the first 47 positive odd integers?

16)_____ 2209 _____

17) The minimum value of $f(x) = \sin x \cos x$ equals _____ .

17)_____ d _____

   (a) 1   **(b)** $\dfrac{1}{2}$ **(c)** -1   **(d)** $-\dfrac{1}{2}$

18) If the straight line $\sqrt{3}\,x - y + m = 0$ is tangent to the circle $x^2 + y^2 - 2x - 2 = 0$, then m equals _____ .

18)_____ c _____

   (a) $\sqrt{3}$ or $-\sqrt{3}$ **(b)** $-\sqrt{3}$ or $3\sqrt{3}$
   (c) $-3\sqrt{3}$ or $\sqrt{3}$ **(d)** $-3\sqrt{3}$ or $3\sqrt{3}$

19) Find $f(2012)$ if $f(x) = a\log_2 x + b\log_3 x + 2$ and $f\left(\dfrac{1}{2012}\right) = 5$.

19)_____ -1 _____

20) What is the maximum value of $2x - y$ if x and y satisfy
$$\begin{cases} x + y \geq 0 \\ x - y + 3 \geq 0 \\ 0 \leq x \leq 3 \end{cases} ?$$

20)_____ 9 _____