

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2013 Junior Examination

(Section I)

NAME: _____

Address: _____

SCHOOL: _____

City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find all real solutions to the equation $|6x + 2| = 20$. 1) $-\frac{11}{3}, 3$

2) Write $\frac{-1 + 3i}{1 + i}$ in the form $a + bi$. 2) $1 + 2i$

3) What is the greatest common divisor of 3840 and 8064? 3) 384

4) What is the remainder when $5x^{10} - 4x^2 + 9x - 13$ is divided by $x - 1$? 4) -3

5) What is the principal square root of $(-2013)^2$? 5) (c)
(a) -2013 (b) ± 2013 (c) 2013 (d) $\pm\sqrt{2013}$

6) Find all values of k such that $4x^2 - 2kx + 1$ is a complete square. 6) $k = \pm 2$

7) Find all real solutions to $4^x - 2^{x+1} - 3 = 0$. 7) $\frac{\ln 3}{\ln 2}$

8) If $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ \frac{2}{x}, & x > 1 \end{cases}$, then what is the value of $f(f(3))$? 8) (d)
(a) $\frac{1}{5}$ (b) 3 (c) $\frac{2}{3}$ (d) $\frac{13}{9}$

9) Find all real solutions to the inequality $2 \leq \log_3 x \leq 5$. 9) $9 \leq x \leq 243$

10) If a point $A : (2a + 3b, -2)$ is symmetric to another point $B : (6, 3a + 2b)$ about the x -axis, then what is $a + b$? 10) $\frac{8}{5}$

(OVER)

11) If $(-4, y_1)$ and $(2, y_2)$ are on the straight line $y = -\frac{1}{2}x + 2$, then which of the following is true?

- (a) $y_1 > y_2$ (c) $y_1 < y_2$
 (b) $y_1 = y_2$ (d) y_1 and y_2 are not comparable

12) Suppose $f(x) = \sqrt[3]{x+4} - 1$. Find $f^{-1}(x)$.

13) Find all real values of x such that $3^{\frac{4}{\log_8 x}} = \frac{1}{27}$.

14) How many distinct ordered pairs, (x, y) , where x and y are integers, are solutions to the inequality $|x| + |y| < 10$?

15) Find the maximum value of $y = \sqrt{3} \sin x \cos x - \sin^2 x$.

16) The solutions to $\frac{x-1}{2x+1} \leq 0$ lie in which interval below?

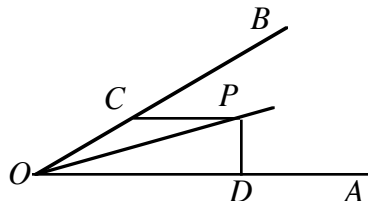
- (a) $(-\frac{1}{2}, 1]$ (c) $(-\infty, \frac{1}{2}) \cup [1, +\infty)$
 (b) $[-\frac{1}{2}, 1)$ (d) $(-\infty, -\frac{1}{2}] \cup [1, +\infty)$

17) Find $\tan 2\alpha$ if $\frac{\sin \alpha + \cos \alpha}{\sin \alpha - \cos \alpha} = \frac{1}{2}$.

18) If $2 < x < 3$, what is the value of $\sqrt{(x-2)^2} + |3-x|$?

19) If a and b are two real solutions to $x^2 + x - 2009 = 0$ then what is the value of $a^2 + 2a + b$?

20) In the figure below, $m\angle AOP = m\angle BOP = 15^\circ$, $\overline{PC} \parallel \overline{OA}$ and $\overline{PD} \perp \overline{OA}$. If $m(\overline{PC}) = 4$, then what is $m(\overline{PD})$?



11) _____ (a) _____

12) $f^{-1}(x) = (x+1)^3 - 4$ _____

13) $\frac{1}{16}$ _____

14) $|8|$ _____

15) $\frac{1}{2}$ _____

16) _____ (a) _____

17) $\frac{3}{4}$ _____

18) _____ 1 _____

19) _____ 2008 _____

20) _____ 2 _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

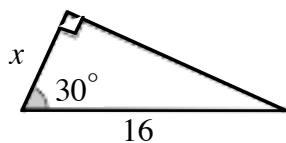
1) For what values of A does the slope of the line passing through the points $(A, -4)$ and $(9, 3)$ equal $\frac{1}{8}$? 1) $A = -47$

2) If $f(x) = x^2 + 1$ and $g(x) = x - 1$ what is $(f \circ g \circ f)(4)$? 2) 257

3) George is 8 years older than Curtis, and Curtis is five times as old as Sue. The sum of their ages is 85. How old is Curtis? 3) 35

4) Suppose $f(x) = 3 - 2x + C$. Find all values of C such that $f(2) = [f(3)]^2$. 4) $C = 2, 5$

5) Find the value of x in the triangle shown at the right. 5) $8\sqrt{3}$



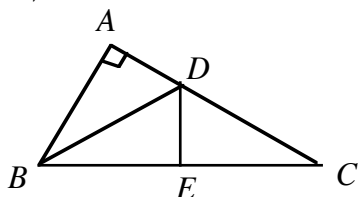
6) If two sides of a triangle are 8 in. and 6 in. long, and the third side has a length which is a real solution of $x^2 - 12x + 20 = 0$, then the perimeter of the triangle is 6) (a)
(a) 24 (b) 26 and 16 (c) 26 (d) 16

7) Express $\sqrt{\sqrt[5]{1024}}$ as a positive integer. 7) 2

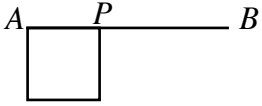
8) If $|x + 1| + |y - 2| = 0$, then what is the value of $x + y$? 8) 1

9) If $f(x) = \log x$, and $f(ab) = 1$, then what is the value of $f(a^2) + f(b^2)$? 9) 2

10) If in a right triangle ABC , $m\angle CBD = m\angle ABD$, $\overline{DE} \perp \overline{BC}$, $m\angle ABC = 60^\circ$ and $m(\overline{BC}) = 10$, then what is the perimeter of $\triangle DEC$? 10) $5\sqrt{3} + 5$



(OVER)

- 11) In the sequence where $a_1 = \frac{3}{5}$, $a_n = 1 - \frac{1}{a_{n-1}}$, $n \geq 2$,
what is the value of a_{2013} ?
- (a) $-\frac{1}{2}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{2}$
- 12) If $f(x) = \sin\left(\frac{x + \varphi}{3}\right)$, $\varphi \in [0, 2\pi)$ is an even function, then what
is the value of φ ?
- 13) If $\sin \alpha - \cos \alpha = \sqrt{2}$ and $\alpha \in (0, \pi)$, then $\sin 2\alpha = ?$
- (a) -1 (b) $-\frac{\sqrt{2}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1
- 14) A point P is randomly selected on a line segment \overline{AB} of length 10.
A square is built upon \overline{AP} . What is the probability that the square
has an area between 25 cm^2 and 49 cm^2 ?
- 
- 15) What is the inverse function of $f(x) = \sqrt{x+1}$ if $x \geq -1$?
- (a) $y = x^2 - 1, x \geq 0$ (c) $y = x^2 - 1, x \geq 1$
(b) $y = x^2 + 1, x \geq 0$ (d) $y = x^2 + 1, x \geq 1$
- 16) What is the value of $2a^2 + 2a + 2001$ if $a^2 + a - 5 = 1$?
- 17) The rectangular coordinate representation of the polar equation
 $r = \frac{1}{\cos \theta + \sin \theta}$ is given by which of the following?
- (a) $y = 1 - x^2$ (c) $y = 1 + x$
(b) $y = 1 + x^2$ (d) $y = 1 - x$
- 18) What is the value of $x^2 + y^2$ if $\left|x - \frac{1}{2}\right| + (2y + 1)^2 = 0$?
- (a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1
- 19) What is the coefficient of the x^3y^{11} term in the expansion of
 $(3x + y)^{14}$?
- 20) Find all real solutions to $|x^2 - 3x - 4| > x + 1$.
- 11) _____ (c) _____
- 12) _____ $\varphi = \frac{3\pi}{2}$ _____
- 13) _____ (a) _____
- 14) _____ $\frac{1}{5}$ _____
- 15) _____ (a) _____
- 16) _____ 2013 _____
- 17) _____ (d) _____
- 18) _____ (b) _____
- 19) _____ 9828 _____
- 20) _____ $(-\infty, 1) \cup (-1, 3) \cup (5, \infty)$ _____

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1) Find the domain of the function $f(x) = \sqrt{\frac{2x}{x^2 - 4}}$. 1) $(-2, 0) \cup (2, \infty)$

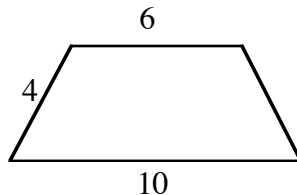
2) Factor the following expression completely: $x^3 - 3x^2 - 25x - 21$. 2) $(x + 1)(x + 7)(x + 3)$

3) Express $(\sqrt{2} - \sqrt{-6})(\sqrt{6} - \sqrt{-2})$ in the form $a + bi$. 3) $-8i$

4) If $f(x) = \begin{cases} x + 1, & x \geq 0 \\ x^2, & x < 0 \end{cases}$, then what is the value of $f(-2)$? 4) (c)

- (a) 1 (b) 2 (c) 4 (d) 5

5) Find the area of the given isosceles trapezoid if its perimeter is 24. 5) $16\sqrt{3}$



6) If z is complex and $z \cdot (1 - i) = 2$, then what is the value of z ? 6) $1 + i$

7) Four consecutive even integers have a sum of 380. What is the smallest of the four integers? 7) 92

8) The foci of the ellipse $\frac{x^2}{7} + \frac{y^2}{16} = 1$ are given by: 8) (a)

- (a) $(0, -3)$ and $(0, 3)$ (c) $(-3, 0)$ and $(3, 0)$
(b) $(0, -4)$ and $(0, 4)$ (d) $(-4, 0)$ and $(4, 0)$

9) Find all real solutions to the system $\begin{cases} x^2 + 2y = 12 \\ 2x - y = -8 \end{cases}$. 9) $(-2, 4)$

10) If $\tan \alpha$ and $\tan \beta$ are the solutions to $x^2 - 3x + 2 = 0$, then what is the value of $\tan(\alpha + \beta)$? 10) (a)

(a) -3 (b) -1 (c) 3 (d) 2

(OVER)

11) Find the domain of $f(x) = \sqrt{1 - 2 \log_6 x}$.

11) $(-\infty, \sqrt{6})$

12) What is the value of $\sqrt[3]{ab}$ if $\sqrt{1 - 4a} + |b + 4| = 0$?

12) -1

13) What is the distance between the points $(-2, 5)$ and $(3, 7)$?

13) $\sqrt{29}$

14) How many real zeros does $f(x) = x^{\frac{1}{2}} - \left(\frac{1}{2}\right)^x$ have?

- (a) 0 (b) 1 (c) 2 (d) 3

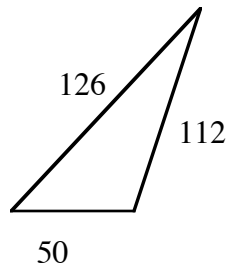
14) (b)

15) What is the center of the circle $x^2 + y^2 - 14x + 8y + 49 = 0$?

- (a) $(-14, 8)$ (b) $(14, -8)$ (c) $(7, -4)$ (d) $(-7, -4)$

15) (c)

16) Find the area of the given triangle.



16) $288\sqrt{94}$

17) Find all solutions to the equation $\sin x + 1 = \cos x$ in the interval $[0, 2\pi]$.

17) $0, \frac{3\pi}{2}, 2\pi$

18) Find all real solutions to $(\log_4 8)(\log_8 6) = \log_2 x$.

18) $\sqrt{6}$

19) What is the volume of a pyramid which has a square base with a side of length 4 and a height of 6?

19) 32

20) Find the maximum value of $y = \sin x - \sqrt{3} \cos x$ if $0 \leq x \leq 2\pi$.

20) 2

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- 1) If the diameter of a circle is doubled, then the area of the circle increases by a factor of ?
(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) 4
1) _____ (d)
- 2) Find the equation of the line (in slope-intercept form) that passes through the points (7, -2) and (-3, 6).
 $y = -\frac{4}{5}x + \frac{18}{5}$
2) _____
- 3) If $M = \{ (x, y) \mid x + y = 2 \}$ and $N = \{ (x, y) \mid x - y = 4 \}$ then what is $M \cap N$?
(a) $x = 3, y = 1$ (c) $\{ 3, -1 \}$
(b) $(3, -1)$ (d) $\{(3, -1)\}$
3) _____ (d)
- 4) Assume $x > 0, y > 0$ and $z > 0$. Eliminate negative exponents and reduce $\left(\frac{x^3 y^{-\frac{2}{5}}}{z^4} \right)^{-2} \left(\frac{z^{-1} \sqrt{y}}{\ln e^{\frac{x}{2}}} \right)^3$ to lowest terms.
 $\frac{8z^5 y^{10}}{x^9}$
4) _____
- 5) What is the 22nd term of the sequence 20, 12, 4, -4, -12, -20, ... ?
5) _____ - 148
- 6) What is the vertex of the parabola $y = 2x^2 + 3x - 9$?
6) _____ $(-\frac{3}{4}, -\frac{81}{5})$
- 7) If $f(x) = \frac{x+3}{x+4}$, find $f^{-1}(x)$.
7) $f^{-1}(x) = \frac{3-4x}{x-1}$
- 8) What is the constant term in the expansion of $\left(\sqrt{x} + \frac{1}{2\sqrt{x}} \right)^4$?
(a) $\frac{1}{4}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) 3
8) _____ (b)
- 9) Find all values of x such that $\frac{x}{x+3} + \frac{6}{x^2-9} = \frac{1}{x-3}$.
9) _____ $x = 1$
- 10) Find the domain of $f(x) = \frac{1}{\ln(x+1)} + \sqrt{4-x^2}$.
10) _____ $(-1, 2]$

(OVER)

11) How many different ways can 8 marbles be placed in a row given that 3 are green, 3 are red, and 2 are black?

11) _____ 560 _____

12) Write $(1 + \sqrt{3}i)^8$ in the form $a + bi$.

12) _____ $-128 + (128\sqrt{3})i$ _____

13) Find all values of k such that $3kx^2 + kx + 7 = 0$ has exactly one real solution.

13) _____ $k = 84$ _____

14) If the sequence $\{a_n\}_{n \geq 1}$ is given by $a_1 = 0$, $a_2 = -|a_1 + 1|$, $a_3 = -|a_2 + 2|$, $a_4 = -|a_3 + 3|$, ..., then what is the value of a_{2013} ?

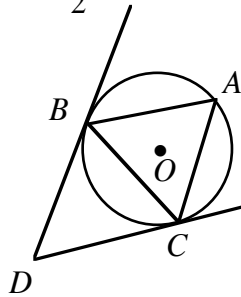
14) _____ (b) _____

(a) -1005 (b) -1006 (c) -1007 (d) -2012

15) What does $\tan x$ equal if $\cos x = -\frac{2}{7}$ and $\frac{\pi}{2} \leq x \leq \pi$?

15) _____ $-\frac{3}{2}\sqrt{5}$ _____

16) \overline{DB} and \overline{DC} are tangent to $\odot O$. If $m\angle D = 46^\circ$, what is $m\angle A$?



16) _____ 67° _____

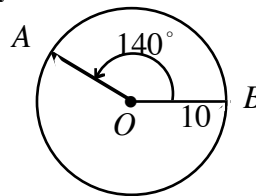
17) What is the area of a square that has a perimeter that is equal to the circumference of a circle with radius 8?

17) _____ $16\pi^2$ _____

18) Evaluate $\cos(\arcsin \frac{4}{5} - \arccos \frac{4}{5})$.

18) _____ $\frac{24}{25}$ _____

19) In the given circle of radius 10, what is the area of sector \widehat{AOB} ?



19) _____ $\frac{350}{9}\pi$ _____

20) What is the minimum value of $x + 2y$ if x and y satisfy

20) _____ (c) _____

$$\begin{cases} x + y \leq 1 \\ x - y \leq 1 \\ x + 1 \geq 0 \end{cases}$$

(a) 3 (b) 1 (c) -5 (d) -6