

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2014 Junior Examination

(Section II)

NAME: _____

Address: _____

SCHOOL: _____

City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) Write $\frac{19}{2-i}$ in the form $a + bi$. 1) _____
- 2) Let $M = \{x \mid (x-1)^2 < 4, x \in \mathbb{R}\}$ and let $N = \{-1, 0, 1, 2, 3\}$. 2) _____
What is $M \cap N$?
- 3) What is the circumference of a circle whose area is 3π in²? 3) _____
- 4) The negation of the statement "All students find the LCCTM Math Contest fun to take." is given by 4) _____
(a) No students find the LCCTM Math Contest fun to take.
(b) Some students find the LCCTM Math Contest fun to take.
(c) At least one student does not find the LCCTM Math Contest fun to take.
(d) (a) and (c).
- 5) If $f(x+2) = \frac{1}{f(x)}$, for any real x , and $f(1) = -5$, what is $f(5)$? 5) _____
- 6) If a complex number z satisfies $(z-3)(2-i) = 5$, then what is the conjugate, \bar{z} , of z ? 6) _____
(a) $2+i$ (b) $2-i$ (c) $5+i$ (d) $5-i$
- 7) If a pair of dice are rolled, what is the probability that the dice show a sum other than 2, 6, or 8? 7) _____
- 8) Find the sum: $3 + \frac{3}{8} + \frac{3}{64} + \frac{3}{512} + \dots$. 8) _____
- 9) Find all real solutions to $3 - \frac{17}{x} + \frac{10}{x^2} \geq 0$. 9) _____
- 10) If $f(x) = 4x + \frac{a}{x}$ for $a, x > 0$ has its smallest value at $x = 3$, 10) _____
then what does a equal?

(OVER)

11) What is $f(f(2))$ if $f(x) = \begin{cases} 2e^x, & x < 2 \\ \log_3(x^2 - 1), & x \geq 2 \end{cases}$? 11) _____

12) Find all real solutions to the equation $\sqrt{\sqrt{8x^2 + 20}} = x$. 12) _____

- 13) If $a = \log_3 6$, $b = \log_5 10$, and $c = \log_7 14$, then 13) _____
- (a) $c > b > a$ (c) $a > c > b$
 (b) $b > c > a$ (d) $a > b > c$

14) If x and y satisfies $\begin{cases} x \geq 1 \\ x - y + 1 \leq 0 \\ 2x - y - 2 \leq 0 \end{cases}$, what is the smallest 14) _____
 value of $x^2 + y^2$?

- 15) A piece of wire 30 inches long is cut into two pieces. One piece is bent into an equilateral triangle and the other is discarded. What is the area of this triangle in terms of x , if the discarded piece of wire was x inches long? 15) _____

- (a) $\frac{\sqrt{3}}{4} \left(10 - \frac{x}{3}\right)^2 \text{ in}^2$ (c) $\frac{1}{2} \left(10 - \frac{x}{3}\right)^2 \text{ in}^2$
 (b) $\frac{\sqrt{3}}{2} \left(10 - \frac{x}{3}\right)^2 \text{ in}^2$ (d) $\frac{3}{4} \left(10 - \frac{x}{3}\right)^2 \text{ in}^2$

16) What is the shortest distance between $(0, 1)$ and a point on the 16) _____
 curve $\begin{cases} x = t \\ y = 2t \end{cases}$?

17) Find all real solutions to $4^x - 3 \cdot 2^{x+2} + 20 = 0$. 17) _____

18) Find $\tan(\alpha + \beta)$ if $\tan \alpha = -\frac{1}{3}$ and $\cos \beta = \frac{\sqrt{5}}{5}$ 18) _____
 for $\alpha, \beta \in \left(-\frac{\pi}{2}, 0\right)$.

19) If $a_n = n \cos \frac{n\pi}{2}$ for $n \geq 1$, what is $a_1 + a_2 + \dots + a_{2014}$? 19) _____

20) If $\begin{cases} x \geq 1 \\ x + y \leq 3 \\ y \geq a(x - 3) \end{cases}$ for $a > 0$, and the smallest value of $2x + y$ 20) _____
 equals 1, what is the value of a ?

- (a) $\frac{1}{4}$ (c) 1
 (b) $\frac{1}{2}$ (d) 2

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the sum of the first 10 prime numbers? 1) _____

2) What does $(f \circ f \circ f)(4)$ equal if $f(x) = 2x + 6$? 2) _____

3) Find all real solutions to $\frac{2}{x-3} + 2 = \frac{12}{x^2-9}$. 3) _____

4) What is the vertex of the parabola $y = x^2 + 10x - 20$? 4) _____

5) Find the value of $(1 - \sqrt{3})^0 + |-\sqrt{2}| - 2\cos 45^\circ + \left(\frac{1}{4}\right)^{-1}$. 5) _____

6) Matthew, Jacob, Robert, and John stand in a line randomly. What is the probability that Matthew and John are next to each other? 6) _____

7) What is the phase shift of $8\cos(3x - 2\pi) + 1$? 7) _____

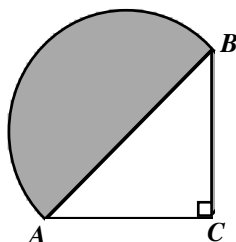
(a) 2π (b) $\frac{2\pi}{3}$ (c) 3 (d) $\frac{1}{3}$

8) What is the smallest distance between P and Q if P is a point on the circle $(x - 3)^2 + (y + 1)^2 = 4$ and Q is a point on the line $x = -3$? 8) _____

(a) 6 (b) 4 (c) 3 (d) 2

9) Find the inverse function, $f^{-1}(x)$, if $f(x) = \frac{5x + 7}{8 - 9x}$. 9) _____

10) What is the length of \overline{AC} in the figure below if $\triangle ABC$ is an isosceles right triangle and the shaded region is a semicircle having an area of $50\pi \text{ m}^2$? 10) _____

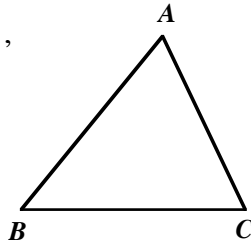


(OVER)

11) Which complex number, z , satisfies $(1 - i)z = 2i$? 11) _____
 (a) $-1 + i$ (b) $-1 - i$ (c) $1 + i$ (d) $1 - i$

12) Find $\sin\theta + \cos\theta$ if $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2}$ when $\frac{\pi}{2} < \theta < \pi$. 12) _____

13) In $\triangle ABC$, $m(\overline{AC}) = 3$, $m(\overline{BC}) = \sqrt{5}$,
 and $\sin C = 2\sin A$. What is $m(\overline{AB})$? 13) _____



14) If $f(x)$ is an odd function with domain all non-zero real numbers,
 and $f(x) = x^2 + \frac{1}{x}$ for $x > 0$, what is $f(-1)$? 14) _____

15) If $a_0 = 7$ and $a_{n+1} = 2a_n + 1$, for $n \geq 0$, then what is a_{50} ? 15) _____
 (a) $2^{52} - 1$ (b) $2^{53} - 1$ (c) $2^{54} - 1$ (d) $2^{55} - 1$

16) What is the domain of $f(x) = \sqrt{\frac{2x^2 - 9x - 18}{x^2 + 4x - 5}}$? 16) _____

17) Find all real solutions to $e^{x \ln 5} = \sqrt{125}$. 17) _____

18) Find the sum: $37 + 39 + 41 + 43 + \dots + 103$. 18) _____

19) If $y = e^x$ and $y = f(x)$ are symmetric about $y = x$, then which of
 following are true? 19) _____

- (a) $f(2x) = e^{2x}$
- (b) $f(2x) = \ln 2 \cdot \ln x, (x > 0)$
- (c) $f(2x) = 2e^{2x}$
- (d) $f(2x) = \ln 2 + \ln x, (x > 0)$

20) If x and y satisfy $\begin{cases} x + 2y \leq 8 \\ 0 \leq x \leq 4 \\ 0 \leq y \leq 3 \end{cases}$, what is the largest value of $x + y$? 20) _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the prime factorization of 31,850? 1) _____

2) Find the equation of the line that is perpendicular to $4x - 7y = 2$ and passes through the point (3, 1). Write your answer in slope-intercept form. 2) _____

3) Factor $ab^2 - 4ab + 4a$ completely. 3) _____

4) Find all real solutions to $\frac{x}{x-2} - \frac{1}{x^2-4} = 1$. 4) _____

5) If the radius of a right circular cylinder is increased by a factor of π , then its volume will increase by what factor? 5) _____
(a) $\frac{\pi}{2}$ (b) π (c) π^2 (d) 2π

6) Find $\frac{f(x+h) - f(x)}{h}$ if $f(x) = \frac{1}{x+1}$. 6) _____

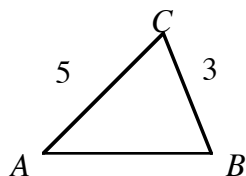
7) Find all solutions to $2 \sin 2t = -\sqrt{3}$ in $[0, 2\pi]$. 7) _____

8) Find all values of k such that $(2, k)$ is 6 units from the point $(-3, 6)$. 8) _____

9) What is the domain of $y = \frac{\log(x+1)}{x}$? 9) _____

10) In $\triangle ABC$, $m(\overline{AC}) = 5$, $m(\overline{BC}) = 3$, and $\sin(\angle BAC) = \frac{1}{3}$. 10) _____

What is the value of $\sin(\angle ABC)$?



(OVER)

11) If $y = f(x) + x^2$ is odd, $f(1) = 1$, and $g(x) = f(x) + 2$, what is $g(-1)$? 11) _____

12) If $\sqrt[7]{\sqrt[8]{x^{\log_3 27}}}$ is rewritten as x^c , what is c ? 12) _____

- (a) $\frac{3}{112}$ (b) $\frac{3}{56}$ (c) $\frac{1}{84}$ (d) $\sqrt{\frac{3}{112}}$

13) The probability of a day being cloudy is 40% and the probability of it being cloudy and windy is 16%. Given that the day is cloudy, what is the probability that it will be windy? 13) _____

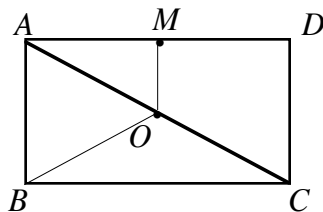
14) For what values of m does $x^2 - 3x + m = 0$ have at least one real solution? 14) _____

15) How many real zeros does $f(x) = 2^x + x^3 - 2$ have in $(0, 1)$? 15) _____

- (a) 0 (b) 1 (c) 2 (d) 3

16) Find the real solutions to $\begin{cases} 3x > x - 2 \\ \frac{x + 1}{3} > 2x \end{cases}$. 16) _____

17) O is the midpoint of the diagonal of rectangle $ABCD$. M is the midpoint of \overline{AD} . If $m(\overline{AB}) = 5$ and $m(\overline{AD}) = 12$, what is the perimeter of $ABOM$? 17) _____



18) If $a > b$, $c < 0$, which of the following must always be true? 18) _____

(a) $a + c > b + c$ (c) $ac > bc$
 (b) $c - a > c - b$ (d) $\frac{a}{c} > \frac{b}{c}$

19) The three cube roots of $8i$ are : 19) _____

- (a) $\sqrt{3} - i, -\sqrt{3} - i$, and $-2i$
 (b) $\sqrt{3} + i, -\sqrt{3} + i$, and $-2i$
 (c) $\sqrt{3} + i, -\sqrt{3} + i$, and $2i$
 (d) $-\sqrt{3} + i, -\sqrt{3} - i$, and $2i$

20) Find all real solutions to $3\left(\frac{2}{3}\right)^{x+3} = \frac{2}{3}\left(\frac{2}{9}\right)^{x+2}$. 20) _____

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- 1) What is $g(1)$ if $f(-1) + g(1) = 2$, $f(1) + g(-1) = 4$, $f(x)$ is odd and $g(x)$ is even? 1) _____
(a) 8 (b) 3 (c) 2 (d) 1
- 2) If $1 + \sqrt{2}i$ is a solution to $x^2 + bx + c = 0$ where b and c are real, then which of the following is true? 2) _____
(a) $b = 2, c = 3$ (c) $b = -2, c = -1$
(b) $b = -2, c = 3$ (d) $b = 2, c = -1$
- 3) Find the exact value of $\csc\left(\arctan\frac{7}{3}\right)$. 3) _____
- 4) If $x^2 - 4x - 1 = 0$, then what is the value of $(2x - 3)^2 - (x + y)(x - y) - y^2$? 4) _____
- 5) If $\left(8, -\frac{5\pi}{4}\right)$ is a point in polar coordinates, what is its rectangular coordinate representation? 5) _____
- 6) A custom frame for a picture is 3 inches wide. The picture inside the frame is 6 inches longer than it is wide. If the area of the picture is 352 square inches, how long is the frame? 6) _____
- 7) Find the range of $f(x) = \begin{cases} \log_{\frac{1}{2}} x, & x \geq 1 \\ 2^x, & x < 1 \end{cases}$. 7) _____
- 8) Find the maximum value of $\sqrt{(3 - a)(a + 6)}$. 8) _____
- 9) The graph of the polar equation $r = \frac{5}{3\cos\theta + 4\sin\theta}$ represents which of the following? 9) _____
(a) a circle (c) a triangle
(b) a line (d) a parabola
- 10) Factor $x^3 + 10x^2 + 17x - 28$ completely. 10) _____

(OVER)

11) What does a equal if the coefficient of x^2 in the expansion of $(1 + ax)(1 + x)^5$ is 5? 11) _____

12) Initially a large pitcher contains 60 ounces of lemonade that is 10% lemon juice. How many ounces of 100% pure lemon juice should be added to the pitcher if the lemonade is to contain 28% lemon juice? 12) _____

13) Find all real solutions to $\left| \frac{2x + 3}{x} \right| \leq 8$. 13) _____

14) Find all real solutions to the system $\begin{cases} x^3 + x = 5y \\ 2x - y = 0 \end{cases}$. 14) _____

15) Rewrite the expression $\frac{5 + x}{\sqrt[3]{x}}$ so that its denominator is rationalized. 15) _____

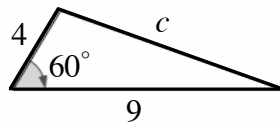
16) What is $\cos \alpha$ if $\sin\left(\frac{5\pi}{2} + \alpha\right) = \frac{1}{5}$? 16) _____

- (a) $-\frac{2}{5}$ (b) $-\frac{1}{5}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

17) If a real number, x , is randomly selected from $[-1, 1]$, what is the probability that $\cos \frac{\pi x}{2}$ is between 0 and $\frac{1}{2}$? 17) _____

- (a) $\frac{1}{3}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

18) Find c in the triangle below. 18) _____



19) Find all real solutions to $\log_6(x - 2) + \log_6(x - 3) = \log_6 12$. 19) _____

20) Find β if $\cos \alpha = \frac{1}{7}$ and $\cos(\alpha - \beta) = \frac{13}{14}$ for $0 < \beta < \alpha < \frac{\pi}{2}$. 20) _____