LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2015 Junior Examination
(Section I)

NAME: _____ **Address:** _____

SCHOOL: _____ **City/ZIP:** _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) If a fair coin is flipped 4 times, what is the probability of of getting exactly 2 heads and 2 tails? **1)**_____

2) Find a function that expresses the area, A, of a square of side length s in terms of its perimeter, p. **2)**_____

3) Compute $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = 2x^2 + 3x$. **3)**_____

4) What is the real part of the complex number $(5 - 2i)^2$? **4)**_____

5) The domain of the function $y = \dfrac{\sqrt{x}}{2x-1}$ is _____ . **5)**_____

(a) $x \geq 0$ **(b)** $x \neq \dfrac{1}{2}$

(c) $x \geq 0$ and $x \neq \dfrac{1}{2}$ **(d)** all real values

6) If set $M = \{0, 1, 2\}$ and set $N = \{x \mid x^2 - 3x + 2 \leq 0\}$, then $M \cap N =$ **6)**_____

(a) $\{ 1 \}$ **(b)** $\{ 2 \}$

(c) $\{ 0, 1 \}$ **(d)** $\{ 1, 2 \}$

7) If $x^2 + 16x + k$ is a complete square, then what is the value of k ? **7)**_____

(a) 64 **(b)** 48 **(c)** 32 **(d)** 16

8) If $f(x) = \begin{cases} x+2 & x < 0 \\ \sqrt{9x} & 0 \leq x \leq 3 \\ x^2 & x > 3 \end{cases}$, then what is the value of **8)**_____

$(f \circ f \circ f \circ f)(-1)$?

9) Find all values of k such that $3x^2 + 7x + k \geq 0$ **9)**_____

10) Find all real solutions to $|3x + 8| \geq 1$. **10)**_____

(OVER)

11) All real solutions to $\begin{cases} x(x+2) > 0 \\ |x| < 1 \end{cases}$ are given by **11)**_____

(a) $\{x \mid -2 < x < -1\}$ **(b)** $\{x \mid -1 < x < 0\}$

(c) $\{x \mid 0 < x < 1\}$ **(d)** $\{x \mid x > 1\}$

12) By how much does the volume of a sphere increase if its radius, r, is increased by 1 unit? Express your answer in terms of r. **12)**_____

13) Given $f(x) = \frac{6}{x} - \log_2 x$, which interval contains the zero point of $f(x)$? **13)**_____

(a) $(0, 1)$ **(b)** $(1, 2)$ **(c)** $(1, 4)$ **(d)** $(4, +\infty)$

14) What is the coefficient of x^2y^7 in the expansion of $(x-y)(x+y)^8$? **14)**_____

15) What is the maximum value attained by **15)**_____

$f(x) = \sin(x + 2\alpha) - 2\sin\alpha\cos(x+\alpha)$?

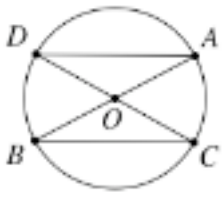
16) Solve for a in $\log_a 3 + \log_a 12 = 2$. Express your answer as as an integer. **16)**_____

17) Find all real solutions to $\sqrt{20 + \sqrt{x}} - 2 = \sqrt[4]{x}$ **17)**_____

18) $\overline{AB}$ and $\overline{CD}$ are two diameters of a circle centered at O. If $m\angle ABC = 30^\circ$, then what is $m\angle BAD$? **18)**_____

(a) 45° **(b)** 60°

(c) 90° **(d)** 30°



19) Find all real solutions to $|x-1| + |x+2| > 5$. **19)**_____

20) Find all real solutions to $e^{4\ln x} - 4^{\log_2(\sqrt{8}\,x)} - 9 = 0$. **20)**_____

LUZERNE COUNTY MATHEMATICS CONTEST

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the value of $\left(\sqrt{2015} - 1\right)^0 + \sqrt{18}\sin 45^\circ - 2^2$? **1)**_____

2) Write $\left(\sqrt{7} + i\sqrt{5}\right)^2$ in the form $a + bi$. **2)**_____

3) If a number is randomly selected from $[-2, 3]$, what is the probability that the number is less than 1? **3)**_____

(a) $\frac{4}{5}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{2}{5}$ **(d)** $\frac{1}{5}$

4) If $A = \frac{M}{2}$, $A = 3T$, and $T = 4H$, what does $M + A + T + H$ equal when $H = 2$? **4)**_____

5) What is the least common multiple of 728 and 676? **5)**_____

6) If the area of a circle is twice its circumference, what is the value of the circle's radius? **6)**_____

7) If $a = \frac{1}{2}$, then what is the value of $\frac{1 + a}{1 - a^2} + \frac{2}{1 - a}$? **7)**_____

8) The rectangular equation for the polar equation $\theta = \frac{\pi}{4}$ is given by **8)**_____

(a) $x^2 + y^2 = \frac{1}{2}$ **(b)** $y = -x$

(c) $y = x$ **(d)** $x^2 + y^2 = \frac{\sqrt{2}}{2}$

9) Reduce $\frac{x^2 + xy - x - y}{x^2 - 1}$ to lowest terms. **9)**_____

10) Express $\frac{\ln 81 - \ln 9}{\ln 3}$ as an integer. **10)**_____

(OVER)

11) Find all real solutions to $\sqrt{\ln x - 2} = \ln x - 2$. **11)**_____

12) If $x = \left(\frac{1}{3}\right)^{-1} + 1$, then what is the value of **12)**_____

$$\left(1 - \frac{1}{x-1}\right) \div \frac{x^2 - 4}{x^2 + 4x + 4}?$$

13) If $f(x)$ satisfies $f(x + \pi) = f(x) + \sin x$ and $f(x) = 0$ **13)**_____
for $0 \le x < \pi$, then $f\left(\frac{23\pi}{6}\right)$ equals _____.

(a) $\frac{1}{2}$ **(b)** $\frac{\sqrt{3}}{2}$

(c) 0 **(d)** $-\frac{1}{2}$

14) What is the maximum value of $y = \cos 2x + 2\sin x$? **14)**_____

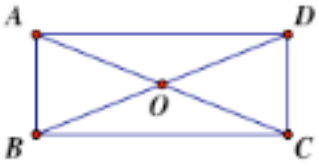
15) If the minute hand on a clock is 8 inches long, how far does the tip of the minute hand travel from 12:00 pm to 2:27 pm? **15)**_____ in._____

16) If $f(x)$ and $g(x)$ are defined on the whole real line, and $f(x)$ is odd and $g(x)$ is even, then which of the following is true? **16)**_____

(a) $f(x) \cdot g(x)$ is even **(b)** $|f(x)| \cdot |g(x)|$ is even

(c) $f(x) \cdot |g(x)|$ is even **(d)** $|f(x) \cdot g(x)|$ is odd

17) In a rectangle $ABCD$ $m(\overline{AC}) = 8$cm and $m\angle AOD = 120^\circ$, then $m(\overline{AB})$ is **17)**_____



(a) $\sqrt{3}$ cm **(b)** 2 cm **(c)** $2\sqrt{3}$ cm **(d)** 4 cm

18) What is the value of the sum below ? **18)**_____

$$1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{9} + \frac{1}{4} + \frac{2}{27} + \frac{1}{8} + \frac{2}{81} + \cdots$$

19) Suppose the first digit of a 2-digit number is represented by x and the second digit by y, where $x, y \ge 0$. Which of the following represents the product of this number and a 2-digit number where the first digit is represented by y and the second digit is represented by x? **19)**_____

(a) $2xy$ **(b)** x^2y^2

(c) $x^2 + y^2 + 2xy$ **(d)** $10x^2 + 10y^2 + 101xy$

20) If x and y satisfy $\begin{cases} y \le x \\ x + y \le 1 \\ y \ge -1 \end{cases}$ and the smallest value and the largest value of $z = 2x + y$ are m and M respectively, then what is the value of $M - m$? **20)**_____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the distance between the points $(4,\ 2)$ and $(-3,\ 7)$? **1)**_____

2) What is the complete factorization of $a^3 - 2a^2 + a$? **2)**_____

3) If $M = \{-1,\ 0,\ 1\}$ and $N = \{0,\ 1,\ 2\}$, then what is $M \cup N$? **3)**_____

4) If a pizza parlor offers 10 toppings, how many different 3 topping pizzas can be made assuming the toppings are unique? **4)**_____

5) What is the largest perfect square less than 800? **5)**_____

6) The graph of $f(\theta) = \cos\theta\tan\theta + \sin\theta$ is symmetric about _____. **6)**_____

(a) the origin. **(b)** the x-axis.
(c) the y-axis. **(d)** none of the above.

7) Express $\dfrac{\log 0.01^{10}}{\sqrt{0.0001}}$ as an integer. **7)**_____

8) If a complex number, z, satisfies $(3 + 4i)z = 25$, then z equals _____. **8)**_____

(a) $3 - 4i$ **(b)** $3 + 4i$
(c) $-3 - 4i$ **(d)** $-3 + 4i$

9) If $f(x) = \ln\left(e^{3x} + 1\right) + ax$ is an even function, then what is the value of a? **9)**_____

10) If the points $(-1,\ y_1)$ and $(2,\ y_2)$ are on the curve $y = \dfrac{3 + 2m}{x}$, and $y_1 > y_2$, then m satisfies which of the following? **10)**_____

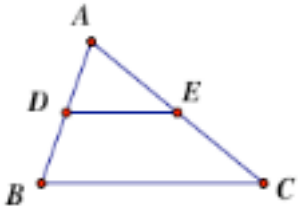
(a) $m < 0$ **(b)** $m > 0$
(c) $m > -\dfrac{3}{2}$ **(d)** $m < -\dfrac{3}{2}$

(OVER)

11) What is the coefficient of the x^4y^3 term in the expansion of $(x + 2y)^7$? **11)**_____

12) How many zeros does the function $f(x) = 2^x \left| \log_{0.5} x \right| - 1$ have? **12)**_____
(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

13) D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively. If $m(\overline{DE}) = 5$, then what is $m(\overline{BC})$? **13)**_____



14) What is the value of $(-2)^{-2} + \left| \sin 30^\circ - 1 \right| + \left(\frac{1}{\pi}\right)^0 + \sqrt{\frac{1}{16}}$? **14)**_____

15) Find *A* and *B* such that $\frac{4x - 26}{x^2 + 2x - 8} = \frac{A}{x + 4} + \frac{B}{x - 2}$. **15)** *A* = _____ *B* = _____

16) When 20 apple trees are planted on an orchard, each tree yields 100 apples. The yield per tree reduces by 2 apples for each additional tree that is planted. What is the maximum yield of the orchard? **16)**_____

17) What is the period of the function $f(x) = \frac{\sqrt{3}}{2}\sin 2x + \cos^2 x$? **17)**_____

18) Find all solutions to $3\sec^2 x - 2\tan^2 x = 4$ in $[0, 2\pi)$. **18)**_____

19) In a geometric sequence $\{a_n\}$, where $a_n > 0$, $n \geq 1$, if $a_2 = 1$ and $a_8 = a_6 + 2a_4$, what is a_6? **19)**_____

20) If a square with side length *s* and an equilateral triangle with side length *l* both have equal areas, then which of the following must be true? **20)**_____

(a) $s = \frac{\sqrt[4]{3}}{2}l$ **(b)** $l = \frac{\sqrt[4]{3}}{2}s$

(c) $s = \frac{4\sqrt{3}}{3}l$ **(d)** $l = \frac{4\sqrt{3}}{3}s$

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) If a die is rolled twice, what is the probability of getting a sum of 5 on the two rolls ? **1)**_____

(a) $\frac{1}{18}$ **(b)** $\frac{1}{9}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{12}$

2) What is the equation of the line, in slope-intercept form, that is parallel to $2x - \pi y = 8$ and passes through the point $\left(\frac{\pi}{2}, 3\right)$? **2)**_____

3) The difference between the squares of two consecutive positive odd integers is 232. What are these integers? **3)**_____

4) What is the value of $\cos 2\theta$ if $\sin\theta + \cos\theta = \frac{1}{5}$ and $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$? **4)**_____

5) The complex number $\frac{7+i}{3+4i}$ is equal to **5)**_____

(a) $1 - i$ **(b)** $-1 + i$

(c) $\frac{17}{25} + \frac{13}{25}i$ **(d)** $-\frac{17}{7} + \frac{25}{7}i$

6) Simplify $\left(\frac{x^{-5}y^{9}}{2x^{4}y^{3}}\right)^{-4}$ by eliminating negative exponents and reducing to lowest terms. **6)**_____

7) What is the constant term in the expansion of $\left(x - \frac{1}{\sqrt{x}}\right)^{6}$? **7)**_____

8) Find all real solutions to $\frac{1}{x-1} = \frac{3}{2x+3}$. **8)**_____

9) What is the value of $\alpha^2 + 4\alpha + \beta$ if α and β are two roots of $x^2 + 3x - 7 = 0$? **9)**_____

10) If $3x + 2y = 1$ and $2x + 3y = 4$, what is the value of $x + 4y$? **10)**_____

(OVER)

11) What is the diameter of a circle whose area is 2π square units? **11)**_____

12) What is the complete factorization of $x^3 + 6x^2 + 11x + 6$? **12)**_____

13) If $a < c < 0 < b$, then which of the following is true? **13)**_____

(a) $abc < 0$ **(b)** $abc = 0$

(c) $abc > 0$ **(d)** undetermined

14) Find the domain of $f(x) = \sqrt{\dfrac{\ln(x-3)}{\sqrt{x+4}}}$. **14)**_____

15) What is a_7 if $a_1 = 2$ and $a_n = na_{n-1}$ for $n \geq 2$? **15)**_____

16) Find all solutions to $\cos 3x = \dfrac{\sqrt{2}}{2}$ in $[0, \pi)$. **16)**_____

17) In a geometric sequence where s_n represents the n^{th}, partial sum of the sequence, if $s_2 = 3$, and $s_4 = 15$, what is s_6? **17)**_____

18) The inverse function of $y = \ln\left(\sqrt[3]{x} + 1\right)$ is _____. **18)**_____

(a) $y = \left(1 - e^x\right)^3, x > -1$

(b) $y = \left(e^x - 1\right)^3, x > -1$

(c) $y = \left(1 - e^x\right)^3, x \in \mathbb{R}$

(d) $y = \left(e^x - 1\right)^3, x \in \mathbb{R}$

19) If $f(x)$ is an odd function on $\mathbb{R}$, and $g(x) = f(x+2)$ is an even function on $\mathbb{R}$, what is $f(8) + f(9)$ if $f(1) = 1$? **19)**_____

20) If x and y satisfy $\begin{cases} x + y - 2 \geq 0 \\ x - y - 2 \leq 0 \\ y \geq 1 \end{cases}$, then the smallest value of $z = x + 2y$ is **20)**_____

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5