

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2015 Junior Examination

(Section I)

NAME: _____

Address: _____

SCHOOL: _____

City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) If a fair coin is flipped 4 times, what is the probability of getting exactly 2 heads and 2 tails? 1) $\frac{3}{8}$
- 2) Find a function that expresses the area, A , of a square of side length s in terms of its perimeter, p . 2) $\frac{p^2}{16}$
- 3) Compute $\frac{f(x+h) - f(x)}{h}$ for $f(x) = 2x^2 + 3x$. 3) $4x + 2h + 3$
- 4) What is the real part of the complex number $(5 - 2i)^2$? 4) 21
- 5) The domain of the function $y = \frac{\sqrt{x}}{2x-1}$ is _____ . 5) c
- (a) $x \geq 0$ (b) $x \neq \frac{1}{2}$
- (c) $x \geq 0$ and $x \neq \frac{1}{2}$ (d) all real values
- 6) If set $M = \{0, 1, 2\}$ and set $N = \{x \mid x^2 - 3x + 2 \leq 0\}$, then $M \cap N =$ 6) d
- (a) $\{1\}$ (b) $\{2\}$
- (c) $\{0, 1\}$ (d) $\{1, 2\}$
- 7) If $x^2 + 16x + k$ is a complete square, then what is the value of k ? 7) a
- (a) 64 (b) 48 (c) 32 (d) 16
- 8) If $f(x) = \begin{cases} x+2 & x < 0 \\ \sqrt{9x} & 0 \leq x \leq 3 \\ x^2 & x > 3 \end{cases}$, then what is the value of $(f \circ f \circ f \circ f)(-1)$? 8) 27
- 9) Find all values of k such that $3x^2 + 7x + k \geq 0$ 9) $k \geq \frac{49}{12}$
- 10) Find all real solutions to $|3x + 8| \geq 1$. 10) $(-\infty, -3] \cup \left[-\frac{7}{3}, \infty\right)$

(OVER)

11) All real solutions to $\begin{cases} x(x+2) > 0 \\ |x| < 1 \end{cases}$ are given by

- (a) $\{x \mid -2 < x < -1\}$ (b) $\{x \mid -1 < x < 0\}$
 (c) $\{x \mid 0 < x < 1\}$ (d) $\{x \mid x > 1\}$

11) _____ *c* _____

12) By how much does the volume of a sphere increase if its radius, r , is increased by 1 unit? Express your answer in terms of r .

12) _____ $4\pi r^2 + 4\pi r + \frac{4\pi}{3}$ _____

13) Given $f(x) = \frac{6}{x} - \log_2 x$, which interval contains the zero point of $f(x)$?

- (a) $(0, 1)$ (b) $(1, 2)$ (c) $(1, 4)$ (d) $(4, +\infty)$

13) _____ *c* _____

14) What is the coefficient of x^2y^7 in the expansion of $(x - y)(x + y)^8$?

14) _____ -20 _____

15) What is the maximum value attained by $f(x) = \sin(x + 2\alpha) - 2\sin\alpha \cos(x + \alpha)$?

15) _____ 1 _____

16) Solve for a in $\log_3 3 + \log_3 12 = 2$. Express your answer as an integer.

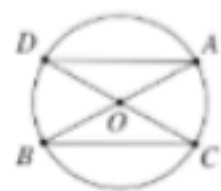
16) _____ 6 _____

17) Find all real solutions to $\sqrt{20 + \sqrt{x}} - 2 = \sqrt[4]{x}$

17) _____ 256 _____

18) \overline{AB} and \overline{CD} are two diameters of a circle centered at O . If $m\angle ABC = 30^\circ$, then what is $m\angle BAD$?

- (a) 45° (b) 60°
 (c) 90° (d) 30°



18) _____ *d* _____

19) Find all real solutions to $|x - 1| + |x + 2| > 5$.

19) _____ $(-\infty, -3) \cup (2, \infty)$ _____

20) Find all real solutions to $e^{4\ln x} - 4^{\log_2(\sqrt{x})} - 9 = 0$.

20) _____ 3 _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) What is the value of $(\sqrt{2015} - 1)^0 + \sqrt{18} \sin 45^\circ - 2^2$? 1) _____ 0 _____
- 2) Write $(\sqrt{7} + i\sqrt{5})^2$ in the form $a + bi$. 2) _____ $2 + 2\sqrt{35}i$ _____
- 3) If a number is randomly selected from $[-2, 3]$, what is the probability that the number is less than 1? 3) _____ b _____
- (a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$
- 4) If $A = \frac{M}{2}$, $A = 3T$, and $T = 4H$, what does $M + A + T + H$ equal when $H = 2$? 4) _____ 82 _____
- 5) What is the least common multiple of 728 and 676? 5) _____ 9,464 _____
- 6) If the area of a circle is twice its circumference, what is the value of the circle's radius? 6) _____ 4 _____
- 7) If $a = \frac{1}{2}$, then what is the value of $\frac{1+a}{1-a^2} + \frac{2}{1-a}$? 7) _____ 6 _____
- 8) The rectangular equation for the polar equation $\theta = \frac{\pi}{4}$ is given by 8) _____ c _____
- (a) $x^2 + y^2 = \frac{1}{2}$ (b) $y = -x$
- (c) $y = x$ (d) $x^2 + y^2 = \frac{\sqrt{2}}{2}$
- 9) Reduce $\frac{x^2 + xy - x - y}{x^2 - 1}$ to lowest terms. 9) _____ $\frac{x+y}{x+1}$ _____
- 10) Express $\frac{\ln 81 - \ln 9}{\ln 3}$ as an integer. 10) _____ 2 _____

(OVER)

11) Find all real solutions to $\sqrt{\ln x - 2} = \ln x - 2$.

11) e^2, e^3

12) If $x = \left(\frac{1}{3}\right)^{-1} + 1$, then what is the value of

12) 2

$$\left(1 - \frac{1}{x-1}\right) + \frac{x^2 - 4}{x^2 + 4x + 4} ?$$

13) If $f(x)$ satisfies $f(x + \pi) = f(x) + \sin x$ and $f(x) = 0$ for $0 \leq x < \pi$, then $f\left(\frac{23\pi}{6}\right)$ equals _____.

13) a

(a) $\frac{1}{2}$

(b) $\frac{\sqrt{3}}{2}$

(c) 0

(d) $-\frac{1}{2}$

14) What is the maximum value of $y = \cos 2x + 2\sin x$?

14) $\frac{3}{2}$

15) If the minute hand on a clock is 8 inches long, how far does the tip of the minute hand travel from 12:00 pm to 2:27 pm?

15) $39\frac{1}{5}\pi$ in.

16) If $f(x)$ and $g(x)$ are defined on the whole real line, and $f(x)$ is odd and $g(x)$ is even, then which of the following is true?

16) b

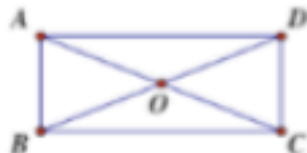
(a) $f(x) \cdot g(x)$ is even

(b) $|f(x)| \cdot |g(x)|$ is even

(c) $f(x) \cdot |g(x)|$ is even

(d) $|f(x) \cdot g(x)|$ is odd

17) In a rectangle $ABCD$ $m(\overline{AC}) = 8\text{cm}$ and $m\angle AOD = 120^\circ$, then $m(\overline{AB})$ is



17) d

(a) $\sqrt{3}$ cm

(b) 2 cm

(c) $2\sqrt{3}$ cm

(d) 4 cm

18) What is the value of the sum below ?

18) 3

$$1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{9} + \frac{1}{4} + \frac{2}{27} + \frac{1}{8} + \frac{2}{81} + \dots$$

19) Suppose the first digit of a 2-digit number is represented by x and the second digit by y , where $x, y \geq 0$. Which of the following represents the product of this number and a 2-digit number where the first digit is represented by y and the second digit is represented by x ?

19) d

(a) $2xy$

(b) x^2y^2

(c) $x^2 + y^2 + 2xy$

(d) $10x^2 + 10y^2 + 101xy$

20) If x and y satisfy $\begin{cases} y \leq x \\ x + y \leq 1 \\ y \geq -1 \end{cases}$ and the smallest value and the largest

20) 6

value of $z = 2x + y$ are m and M respectively, then what is the value of $M - m$?

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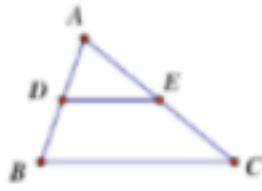
- 1) What is the distance between the points $(4, 2)$ and $(-3, 7)$? 1) $\sqrt{74}$
- 2) What is the complete factorization of $a^3 - 2a^2 + a$? 2) $a(a - 1)^2$
- 3) If $M = \{-1, 0, 1\}$ and $N = \{0, 1, 2\}$, then what is $M \cup N$? 3) $\{-1, 0, 1, 2\}$
- 4) If a pizza parlor offers 10 toppings, how many different 3 topping pizzas can be made assuming the toppings are unique? 4) 120
- 5) What is the largest perfect square less than 800? 5) 784
- 6) The graph of $f(\theta) = \cos \theta \tan \theta + \sin \theta$ is symmetric about _____ 6) a
- (a) the origin. (b) the x -axis.
(c) the y -axis. (d) none of the above.
- 7) Express $\frac{\log 0.01^{10}}{\sqrt{0.0001}}$ as an integer. 7) -2,000
- 8) If a complex number, z , satisfies $(3 + 4i)z = 25$, then z equals _____ 8) a
- (a) $3 - 4i$ (b) $3 + 4i$
(c) $-3 - 4i$ (d) $-3 + 4i$
- 9) If $f(x) = \ln(e^{3x} + 1) + ax$ is an even function, then what is the value of a ? 9) $\frac{3}{2}$
- 10) If the points $(-1, y_1)$ and $(2, y_2)$ are on the curve $y = \frac{3 + 2m}{x}$, and $y_1 > y_2$, then m satisfies which of the following? 10) d
- (a) $m < 0$ (b) $m > 0$
(c) $m > -\frac{3}{2}$ (d) $m < -\frac{3}{2}$

(OVER)

11) What is the coefficient of the x^4y^3 term in the expansion of $(x + 2y)^7$? 11) 280

12) How many zeros does the function $f(x) = 2^{\lfloor \log_{0.5} x \rfloor} - 1$ have? 12) b
 (a) 1 (b) 2 (c) 3 (d) 4

13) D and E are the midpoints of \overline{AB} and \overline{AC} respectively. If $m(\overline{DE}) = 5$, then what is $m(\overline{BC})$? 13) 10



14) What is the value of $(-2)^{-2} + |\sin 30^\circ - 1| + \left(\frac{1}{\pi}\right)^{\pi} + \sqrt{\frac{1}{16}}$? 14) 2

15) Find A and B such that $\frac{4x - 26}{x^2 + 2x - 8} = \frac{A}{x + 4} + \frac{B}{x - 2}$. 15) $A = 7$ $B = -3$

16) When 20 apple trees are planted on an orchard, each tree yields 100 apples. The yield per tree reduces by 2 apples for each additional tree that is planted. What is the maximum yield of the orchard? 16) 2,450

17) What is the period of the function $f(x) = \frac{\sqrt{3}}{2}\sin 2x + \cos^2 x$? 17) π

18) Find all solutions to $3\sec^2 x - 2\tan^2 x = 4$ in $[0, 2\pi)$. 18) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

19) In a geometric sequence $\{a_n\}$, where $a_n > 0$, $n \geq 1$, if $a_2 = 1$ and $a_5 = a_3 + 2a_4$, what is a_6 ? 19) 4

20) If a square with side length s and an equilateral triangle with side length l both have equal areas, then which of the following must be true? 20) a

- (a) $s = \frac{\sqrt{3}}{2}l$ (b) $l = \frac{\sqrt{3}}{2}s$
 (c) $s = \frac{4\sqrt{3}}{3}l$ (d) $l = \frac{4\sqrt{3}}{3}s$

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) If a die is rolled twice, what is the probability of getting a sum of 5 on the two rolls? 1) _____ b _____

- (a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

2) What is the equation of the line, in slope-intercept form, that is parallel to $2x - \pi y = 8$ and passes through the point $\left(\frac{\pi}{2}, 3\right)$? 2) _____ $y = \frac{2}{\pi}x + 2$ _____

3) The difference between the squares of two consecutive positive odd integers is 232. What are these integers? 3) _____ $57, 59$ _____

4) What is the value of $\cos 2\theta$ if $\sin\theta + \cos\theta = \frac{1}{5}$ and $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$? 4) _____ $-\frac{7}{25}$ _____

5) The complex number $\frac{7+i}{3+4i}$ is equal to 5) _____ a _____

- (a) $1 - i$ (b) $-1 + i$
(c) $\frac{17}{25} + \frac{13}{25}i$ (d) $-\frac{17}{7} + \frac{25}{7}i$

6) Simplify $\left(\frac{x^{-5}y^3}{2x^4y^2}\right)^{-4}$ by eliminating negative exponents and reducing to lowest terms. 6) _____ $\frac{16x^{36}}{y^{24}}$ _____

7) What is the constant term in the expansion of $\left(x - \frac{1}{\sqrt{x}}\right)^5$? 7) _____ 15 _____

8) Find all real solutions to $\frac{1}{x-1} = \frac{3}{2x+3}$. 8) _____ $x = 6$ _____

9) What is the value of $\alpha^2 + 4\alpha + \beta$ if α and β are two roots of $x^2 + 3x - 7 = 0$? 9) _____ 4 _____

10) If $3x + 2y = 1$ and $2x + 3y = 4$, what is the value of $x + 4y$? 10) _____ 7 _____

(OVER)

11) What is the diameter of a circle whose area is 2π square units?

11) $2\sqrt{2}$

12) What is the complete factorization of $x^3 + 6x^2 + 11x + 6$?

12) $(x + 1)(x + 2)(x + 3)$

13) If $a < c < 0 < b$, then which of the following is true?

- (a) $abc < 0$
- (b) $abc = 0$
- (c) $abc > 0$
- (d) undetermined

13) c

14) Find the domain of $f(x) = \sqrt{\frac{\ln(x-3)}{\sqrt{x+4}}}$.

14) $[4, \infty)$

15) What is a_7 if $a_1 = 2$ and $a_n = na_{n-1}$ for $n \geq 2$?

15) $10,080$

16) Find all solutions to $\cos 3x = \frac{\sqrt{2}}{2}$ in $[0, \pi)$.

16) $\frac{\pi}{12}, \frac{3\pi}{4}, \frac{7\pi}{12}$

17) In a geometric sequence where s_n represents the n^{th} partial sum of the sequence, if $s_2 = 3$, and $s_4 = 15$, what is s_6 ?

17) 63

18) The inverse function of $y = \ln(\sqrt[3]{x} + 1)$ is _____.

18) d

- (a) $y = (1 - e^x)^3, x > -1$
- (b) $y = (e^x - 1)^3, x > -1$
- (c) $y = (1 - e^x)^3, x \in \mathbb{R}$
- (d) $y = (e^x - 1)^3, x \in \mathbb{R}$

19) If $f(x)$ is an odd function on \mathbb{R} , and $g(x) = f(x + 2)$ is an even function on \mathbb{R} , what is $f(8) + f(9)$ if $f(1) = 1$?

19) 1

20) If x and y satisfy $\begin{cases} x + y - 2 \geq 0 \\ x - y - 2 \leq 0 \\ y \geq 1 \end{cases}$, then the smallest value of $z = x + 2y$ is

20) b

- (a) 2
- (b) 3
- (c) 4
- (d) 5