

Luzerne County Mathematics Contest

Luzerne County Council of Teachers of Mathematics

Wilkes University – 2023 Junior Examination

(Section 1)

NAME: _____ ADDRESS: _____

SCHOOL: _____ CITY/ZIP: _____

TELEPHONE: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the value of the following expression?

$$2 + 16 \div (6 + 2) - 3 \cdot 2$$

1) _____

2) If $2^n = 8^{20}$, what is the value of n ?

2) _____

3) What is the probability of rolling a sum of 7 on two fair, six-sided dice?

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

3) _____

4) What integer is equal to the expression $\sqrt{\frac{\sqrt{81} + \sqrt{81}}{2}}$?

4) _____

5) A semi-circle has radius 10. What is the perimeter of the semi-circle?

5) _____

6) Write the following complex expression in the standard form $a + bi$.

$$\left(\frac{4 - i}{1 + i} - \frac{2i}{2 + i} \right) 4i$$

6) _____

7) What is the sum of the digits in the decimal form of $2^{2022} \cdot 5^{2023}$?

7) _____

8) The following system of equations has a unique solution (x, y) . Find the value of $3x + 2y$.

$$\begin{cases} 4^x 8^y = 32 \\ x - y = 10 \end{cases}$$

8) _____

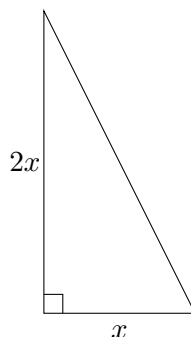
(a) 5 (b) 12 (c) 15 (d) 21

9) Over its domain, what integer is equal to the following trigonometric expression?

$$\frac{\tan^2(t)}{1 - \sec^2(t)}$$

9) _____

10) What is the length of the hypotenuse of the right triangle shown below if its area is 400?



10) _____

(OVER)

11) Find the domain of the following real-valued function. Give your answer in interval notation.

$$f(x) = \frac{\sqrt{x-4}}{\sqrt{x-6}} + |x-5|$$

11) _____

12) A jar contains \$6.65 made up of nickels (value of five cents), dimes (value of ten cents), and quarters (value of twenty five cents). There are the same number of nickels as dimes. If there are 70 total coins, how many more dimes are there than quarters?

12) _____

- (a) 23 (b) 25 (c) 27 (d) 29

13) Let $P(x)$ be a polynomial that has $x-1$, x^2-1 , x^3-1 , and x^4-1 as factors. What is the smallest possible degree of $P(x)$?

13) _____

14) The length of a rectangle exceeds its width by 3 feet. If the perimeter of the rectangle is 46 feet, what is the length, in feet?

14) _____

- (a) 7 (b) 9 (c) 11 (d) 13

15) What is the distance between the minimum and maximum solutions to the following inequality?

15) _____

$$|x| + |3x - 10| \leq 18$$

16) The following expression is equal to what positive integer?

$$\sqrt{\sqrt{\sqrt{\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}}}}}}$$

16) _____

17) What is the area of the circle defined by the equation $x^2 + 6x + y^2 - 4y - 3 = 0$?

17) _____

- (a) 4π (b) 8π (c) 16π (d) 256π

18) How many *distinct* rearrangements of the word **ALPACA** can you make?

18) _____

19) If $\sin \theta = \frac{1}{3}$ and θ is an acute angle, what is the value of $\cos 4\theta$?

19) _____

20) What is the remainder when 2^{2023} is divided by 7?

20) _____

- (a) 1 (b) 2 (c) 4 (d) 6

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1)

1) -2

2)

2) 60

3)

3) b

4)

4) 3

5)

5) $10\pi + 20$

6)

6) $\frac{66}{5} + \frac{22}{5}i$

7)

7) 5

8)

8) c

9)

9) -1

10)

10) $20\sqrt{5}$

(OVER)

11)

11) (6, \infty)

12)

12) a

13)

13) 6

14)

14) d

15)

15) 9

16)

16) 4

17)

17) c

18)

18) 120

19)

19) 17/81

20)

20) b

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(Section 2)

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What integer is equal to the following expression?

$$\frac{\log_2 \left(\frac{1}{256} \right)}{\log_9 \left(\frac{1}{81} \right)}$$

1) _____

2) A high school basketball player makes a free throw with a probability of 0.7. Assuming each free throw is an independent event, what is the probability the player misses all their next three free throws?

2) _____

(a) 0.027 (b) 0.343 (c) 0.657 (d) 0.973

3) Find all real solutions to the equality below.

$$15 - x = 3 - \frac{1}{10 - x}$$

3) _____

4) What integer is equal to $\left(5^{20000/(10000^{-1})} \right)^{10000^{-2}}$?

4) _____

5) Define $f(x) = 2x^2 + 6x - 7$ on the interval $[0, \infty)$. What is the value of $f^{-1}(13)$?

5) _____

6) Find all real solutions to the equality below.

$$2^{2x-5} \cdot 4^{3x-4} = 8^{2x+13}$$

6) _____

7) Two legs of a right triangle, x and y , satisfy $\frac{x+y}{2} = 5$ and $\sqrt{xy} = 3$. What is the length of the hypotenuse of the triangle?

7) _____

8) A line through the points $(m, -9)$ and $(7, m)$ has slope m . Find the value of m .

8) _____

9) The first and fifth terms of an arithmetic sequence are -3 and 17 , respectively. Find the 50th term.

9) _____

(a) 193 (b) 242 (c) 247 (d) 377

10) Find the positive real solution to the equation $2x^2 - 5x^{\frac{4}{3}} - 3x^{\frac{2}{3}} = 0$.

10) _____

(a) $\sqrt{3}$ (b) $\frac{1}{3}$ (c) 1 (d) $3\sqrt{3}$

(OVER)

11) If θ is in Quadrant II and $\sin \theta = \frac{1}{3}$, what is the value of $\tan \theta$? 11) _____

12) For what value of b will the following system of equations have an infinite number of solutions?

$$\begin{aligned}x - 2y + z &= 5 \\ 2x + 3y - 7z &= 1 \\ 3x + 8y - 15z &= b\end{aligned}$$

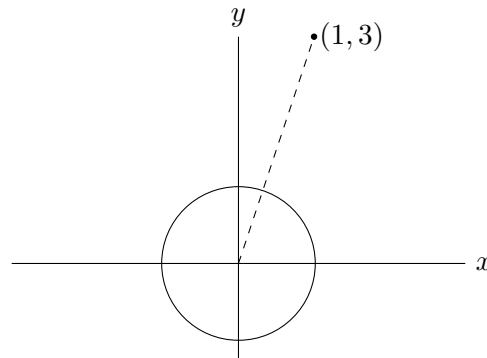
12) _____

13) Suppose f is a function satisfying $f(x) + f(1-x) = 10$ and $f(1+x) = 4 + f(x)$ for all real numbers x . What is the value of $f(100) + f(-100)$? 13) _____

14) What is the slope of the line that bisects the acute angle given by the lines $y = 0$ and $y = x$? 14) _____

- (a) $\frac{1}{2}$ (b) $\frac{\sqrt{2}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{2} - 1$

15) Find the point on the unit circle that is closest to the point $(1, 3)$.



15) _____

16) If the values of A and B make the equation $\frac{35x - 29}{x^2 - 3x + 2} = \frac{A}{x - 1} + \frac{B}{x - 2}$ true for all values of x for which it is defined, find $A + B$. 16) _____

17) What is the sum of the geometric series shown below?

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \dots$$

17) _____

18) If $\sin \theta + \cos \theta = \frac{1}{2}$, what is the value of $\sin 2\theta$? 18) _____

- (a) $-\frac{3}{4}$ (b) $-\frac{3}{8}$ (c) 0 (d) $\frac{3}{8}$

19) For what number base b does the following equality hold? 19) _____

$$(120)_b = 5 \cdot b$$

20) Given two intersecting circles $(x-2)^2 + (y-1)^2 = 4$ and $(x-3)^2 + (y-4)^2 = 9$, what is the equation of the line passing through the points of intersection? 20) _____

- (a) $2x + 6y = 15$ (b) $2x - 6y = 13$ (c) $6x - 2y = 11$ (d) $6x + 2y = 17$

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1)

1) 4

2)

2) a

3)

3) 11

4)

4) 25

5)

5) 2

6)

6) 26

7)

7) $\sqrt{82}$

8)

8) 3

9)

9) b

10)

10) d

(OVER)

11)

$$11) \underline{\underline{-\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}}}$$

12)

$$12) \underline{\underline{-3}}$$

13)

$$13) \underline{\underline{6}}$$

14)

$$14) \underline{\underline{d}}$$

15)

$$15) \underline{\underline{\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)}}$$

" "

$$\underline{\underline{\left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10}\right)}}$$

16)

$$16) \underline{\underline{35}}$$

17)

$$17) \underline{\underline{3}}$$

18)

$$18) \underline{\underline{a}}$$

19)

$$19) \underline{\underline{3}}$$

20)

$$20) \underline{\underline{a}}$$

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What rational number is exactly halfway between $\frac{1}{8}$ and $\frac{1}{10}$? 1) _____

2) If $x + 2y = 2x + y = 84$, what is the value of $x + y$? 2) _____

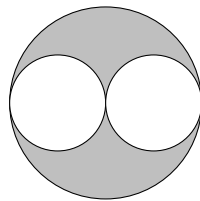
3) The equation $4x + y - 2z = 7$ has infinitely many integer solutions. Which of the following cannot be a value of y for any integer values of x and z ? 3) _____

(a) -1 (b) 1 (c) 2 (d) 5

4) Two fair, 6-sided dice are rolled. What is the probability that their product is an odd number? 4) _____

5) What integer is equal to the expression $\left(\sqrt{4 + \sqrt{4}}\right)^4$? 5) _____

6) In the diagram below, two small circles of radius 1 are tangent to each other and to the larger outer circle. What is the area of the shaded region?



6) _____

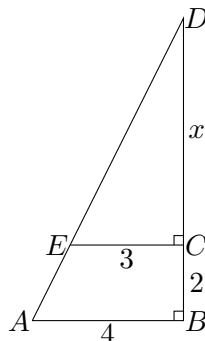
7) Which of the following is equal to the value of $\sqrt{\frac{(5+i)(2-2i)}{3-2i}}$? 7) _____

(a) 2 (b) -2 (c) $2 + i$ (d) $2 - i$

8) Find all real solutions to $\frac{1}{x^2} - \frac{1}{x} = 2$. 8) _____

9) Find all complex numbers that satisfy $|z| + z = 2 + i$. 9) _____

10) What is the length of line segment \overline{CD} in the figure below?



10) _____

(OVER)

11) If θ is in Quadrant III and $\sin(\theta) = -\frac{5}{13}$, what is the absolute value of $\cos\left(\frac{\theta}{2}\right)$? 11) _____

12) Suppose x and y satisfy $\begin{cases} x^2 + y^2 = 20 \\ x + y = 10 \end{cases}$. What integer equals the following expression? 12) _____

$$\frac{1}{\frac{1}{x} + \frac{1}{y}}$$

13) A point (x, y) is 5 units from the origin on the line $y = -2x$. Find x^2y^2 . 13) _____

(a) 100 (b) 400 (c) 500 (d) 2500

14) You have a bag containing 20 marbles. Exactly one of these marbles is yellow. If you randomly draw three marbles from the bag without replacement, what is the probability that you will select the yellow marble? 14) _____

15) What point on the line $y = 2x + 3$ is closest to the point $(-5, 3)$? 15) _____

16) What is the smallest digit d that will make the following number divisible by 6? 16) _____

$$111,278,314,10d$$

17) What is the sum of the first 101 terms of the sequence shown below? 17) _____

$$1, 4, 7, 10, 13, 16, \dots, (3n - 2), \dots$$

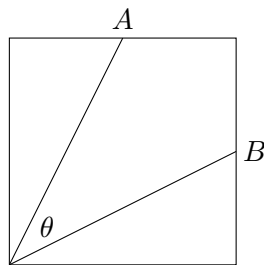
18) How many zeros does the number $30!$ end with? 18) _____

(a) 3 (b) 7 (c) 15 (d) 30

19) How many solutions does the equation $(2 \sin x - 1)(\tan x - 1) = 0$ have in the interval $0 \leq x \leq 2\pi$? 19) _____

(a) 2 (b) 3 (c) 4 (d) 5

20) In the unit square shown below, A and B are midpoints of their respective sides. What is the value of $\cos \theta$? 20) _____



(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) _____ $\frac{9}{80}$

2) _____ 56

3) _____ C

4) _____ $\frac{1}{4}$

5) _____ 36

6) _____ 2π

7) _____ a

8) _____ $-1, \frac{1}{2}$

9) _____ $\frac{3}{4} + i$

10) _____ 6

(OVER)

11)

11) $\frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$

12)

12) 4

13)

13) a

14)

14) $\frac{3}{20}$

15)

15) $(-1, 1)$

16)

16) 4

17)

17) 15251

18)

18) 7

19)

19) c

20)

20) d

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NAME: _____ ADDRESS: _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What integer is equal to $-\log_5([\log_{32}(2)]^3)$? 1) _____

2) Find all real solutions to the equality below.

$$\frac{2^{2023} + 2^a}{2^{2022}} = 66 \quad 2) \text{ _____}$$

3) Over its domain, what integer is equal to the following trigonometric expression?

$$\tan^2(\theta) (\csc^2(\theta) - 1) \quad 3) \text{ _____}$$

4) What rational number in lowest terms is equal to the following expression?

$$2 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3}}} \quad 4) \text{ _____}$$

5) A cone has a height six times its radius. The volume of the cone is 128π . What is the diameter of the cone?

(a) 4 (b) $8\sqrt[3]{9}$ (c) 8 (d) $4\sqrt[3]{9}$ 5) _____

6) How many distinct real solutions are there to the equation

$$(x + 8)^4 = (2x + 16)^2? \quad 6) \text{ _____}$$

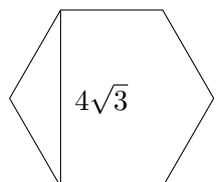
7) Which of the following is equal to $1 - 2 + 3 - 4 + \dots - 98 + 99$?

(a) -50 (b) -49 (c) 0 (d) 50 7) _____

8) If (a, b) and (c, d) are the two intersection points of the line $y = 2x + 1$ and the circle centered at $(1, -1)$ with radius 4, what is $a + b + c + d$?

(a) $-\frac{8}{5}$ (b) $\frac{18}{5}$ (c) $\frac{4\pi}{11}$ (d) $\frac{40\pi}{11}$ 8) _____

9) Compute the perimeter of the regular hexagon shown below.



9) _____

10) If $\sqrt{x} + \sqrt{x+7} = 7$, what is the value of $\frac{1}{2}\sqrt{x-5} + \sqrt{x+7}$? 10) _____

(OVER)

11) Five positive integers have a median of 3 and a *unique* mode of 6. What is their sum? 11) _____

12) If the length of a rectangle is increased by 20% and the width by 25%, by what percentage would the area of the rectangle increase? 12) _____

13) Which of the following is equal to $\tan\left(\frac{\pi}{8}\right)$? 13) _____

- (a) $\sqrt{2} + 1$ (b) $\sqrt{2} - 1$ (c) $\sqrt{3} + 1$ (d) $\sqrt{3} - 1$

14) If you randomly select two balls from an urn that contains 2 red balls and 3 blue balls without replacement, what is the probability of getting a red ball and a blue ball? 14) _____

15) If x and y are positive integers satisfying $x + y = 31$, which of the following is the maximum value of xy ? 15) _____

- (a) 240 (b) 238 (c) 255 (d) 248

16) For what number base b does $(144)_b = (321)_{b-3}$? 16) _____

17) What real number, x , is equal to the following continued fraction expression? 17) _____

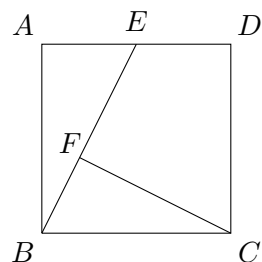
$$x = 2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \dots}}}$$

18) How many *distinct* rearrangements of the word **PEPPER** can you make? 18) _____

19) What is the value of $\sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right) + \sin\left(\frac{3\pi}{2}\right) + \dots + \sin\left(\frac{2023\pi}{2}\right)$? 19) _____

- (a) -1 (b) 0 (c) 1 (d) 2

20) In the figure shown below, $ABCD$ is a 2×2 square, E is the midpoint of AD , and CF is perpendicular to BE . Find the area of quadrilateral $CDEF$.



20) _____

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1)

1) 3

2)

2) 2028

3)

3) 1

4)

4) $\frac{55}{24}$

5)

5) C

6)

6) 3

7)

7) d

8)

8) a

9)

9) 24

10)

10) 5

(OVER)

11)

11) 18

12)

12) 50%

13)

13) b

14)

14) $\frac{3}{5}$

15)

15) a

16)

16) 9

17)

17) $\sqrt{5}$

18)

18) 60

19)

19) b

20)

20) $\frac{11}{5}$