

Luzerne County Mathematics Contest

Luzerne County Council of Teachers of Mathematics

Wilkes University – 2024 Junior Examination

(Section 1)

NAME: _____ ADDRESS: _____

SCHOOL: _____ CITY/ZIP: _____

TELEPHONE: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) How many common factors are shared by 60 and 72? 1) _____

2) Suppose a rectangle has height twice its width and a perimeter of 48. What is its width? 2) _____

3) What is the x -intercept of the line that passes through the points (5, 3) and (11, 2)? 3) _____

4) In the following equation, what integer is equal to x ? 4) _____

$$10^x \cdot 100^{2x} = 1000^5$$

5) If $2^{2024} - 2^{2023} - 2^{2022} + 2^{2021} = k \cdot 2^{2021}$, what is the value of k ? 5) _____

(a) 1 (b) 2 (c) 3 (d) 4

6) On a standard clock, what is the angle (in degrees) between the hands when it is 4:30? 6) _____

7) If the radius of a circle is increased by 100%, the area is increased by: 7) _____

(a) 100% (b) 200% (c) 300% (d) 10000%

8) How many distinct ways can four people stand in a line? 8) _____

9) What is the value of a in the equation below?

$$\frac{1}{\log_2(a)} + \frac{1}{\log_3(a)} + \frac{1}{\log_4(a)} = 1$$

9) _____

(a) 9 (b) 12 (c) 18 (d) 24

10) A triangle with side lengths in the ratio 3:4:5 is inscribed in a circle with radius 5. What is the area of the triangle? 10) _____

(OVER)

11) What integer is equal to the expression below?

$$\sqrt{6\sqrt{6\sqrt{6\sqrt{\dots}}}}$$

11) _____

12) What is the value of y in the system of equations below?

$$x + y + z = 22$$

$$x - y - z = 32$$

$$x - y + z = 42$$

12) _____

13) Find the positive integer n satisfying the equation below.

$$(n + 1)! + (n + 2)! = 143n!$$

13) _____

14) Which of the following angles satisfies $\cot \theta \cos \theta < 0$?

(a) 400° (b) $\frac{36\pi}{7}$ (c) -2024° (d) $-\frac{\pi}{2}$

14) _____

15) The polynomials $x^2 - 3x + 2$ and $x^2 - 5x + k$ have one root in common. What is the sum of the possible values of k ?

(a) 3 (b) 4 (c) 6 (d) 10

15) _____

16) Express i^{237} as a complex number in standard form: $a + bi$.

16) _____

17) The smallest positive value of x for which the function $f(x) = x \cos x - \sqrt{3}x \sin x$ is zero is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

17) _____

18) Real numbers x and y satisfy the equation

$$x^2 + y^2 = 10x - 6y - 34.$$

18) _____

What is the value of $x + y$?

19) How many real numbers in the range $0 \leq x \leq 2\pi$ satisfy the equation

$$2 \cos^2(x) + \sin(x) - 1 = 0?$$

19) _____

20) A box contains 2 red, 2 green, and 2 yellow marbles. Carol takes 2 marbles from the box at random, and then Claudia takes 2 of the remaining marbles. If Cheryl takes the last two marbles from the box, what is the probability that they are the same color?

20) _____

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- | | |
|-----|---------------|
| 1) | 1) <u>6</u> |
| 2) | 2) <u>8</u> |
| 3) | 3) <u>23</u> |
| 4) | 4) <u>3</u> |
| 5) | 5) <u>C</u> |
| 6) | 6) <u>45°</u> |
| 7) | 7) <u>C</u> |
| 8) | 8) <u>24</u> |
| 9) | 9) <u>d</u> |
| 10) | 10) <u>24</u> |

(OVER)

11)

11) 6

12)

12) -10

13)

13) 10

14)

14) b

15)

15) d

16)

16) 0+10

17)

17) a

18)

18) 2

19)

19) 3

20)

20) 1/5

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(Section 2)

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What integer is equal to $999^2 - 998^2$? 1) _____

2) A rectangular box has length 30, width 24, and height 20. What is ratio of its volume to its surface area? 2) _____

3) What is the value of the expression $\frac{(3!)!}{4!}$? 3) _____

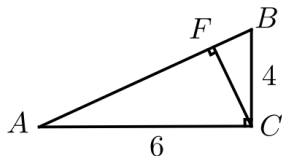
4) Compute the value of the following sum. 4) _____
$$-1 + 2 - 3 + 4 - 5 + 6 - \cdots - 45 + 46$$

5) Find all real x for which $2\log_3(x) + 2\log_9(x) = 9$. 5) _____

6) A group of seven people are waiting for a taxi. The taxi only holds four people. In how many ways can they pick which four people get in the taxi? 6) _____
(a) 28 (b) 35 (c) 840 (d) 21

7) The following trigonometric expression reduces to what integer over its domain? 7) _____
$$\csc^2(\theta) - \cot^2(\theta)$$

8) In the right triangle $\triangle ABC$ pictured below, $\overline{CF}$ is perpendicular to $\overline{AB}$. Find the length of $\overline{CF}$.



(a) $\frac{\sqrt{13}}{13}$ (b) $\frac{2\sqrt{13}}{13}$ (c) $\frac{6\sqrt{13}}{13}$ (d) $\frac{12\sqrt{13}}{13}$ 8) _____

9) Suppose that $f(x+3) = 3x^2 + 7x + 4$ and $f(x) = ax^2 + bx + c$. What is the value of $a + b + c$? 9) _____

10) What is the range of the function $f(x) = \sin(x) + 100$? Give your answer in interval notation. 10) _____

(OVER)

11) The perimeter of a right triangle is 42 and the sum of squares of its sides is 722. Determine the area of the right triangle.

11) _____

- (a) 100 (b) 64 (c) 81 (d) 42

12) Evaluate the following expression.

$$\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} \cdots + \log \frac{98}{99} + \log \frac{99}{100}$$

12) _____

- (a) 0 (b) 2 (c) -2 (d) -1

13) What is the area of $\triangle ABC$ with vertices $A(1, 3)$, $B(1, -5)$, and $C(7, -8)$?

13) _____

14) Point (x, y) is 8 units from the origin on the line $y = -x$. Find the value of $|xy|$.

14) _____

15) How many positive integers x satisfy the following inequality?

$$\log(x - 40) + \log(60 - x) < 2$$

15) _____

- (a) 10 (b) 18 (c) 19 (d) 20

16) Find $\cot(\theta)$ if $\cos(\theta) = \frac{3}{4}$ and θ is in Quadrant I.

16) _____

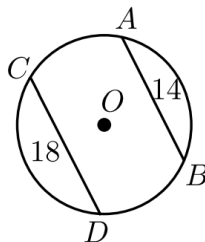
17) Suppose that $\sin(a) + \sin(b) = \sqrt{\frac{5}{3}}$ and $\cos(a) + \cos(b) = 1$. What is the value of $\cos(a - b)$?

17) _____

18) How many subsets of $\{2, 3, 4, 5, 6, 7\}$ contain at least one prime number?

18) _____

19) In the figure shown, the length of $\overline{AB}$ is 14, the length of $\overline{CD}$ is 18, $\overline{AB}$ is parallel to $\overline{CD}$, and the distance between them is 8. What is the length of the radius of the circle?



19) _____

- (a) 8 (b) $\sqrt{85}$ (c) 9 (d) $\sqrt{95}$

20) If $f(x) = \frac{2x}{x+5}$ for $x \neq -5$, find all real solutions to

20) _____

$$f(f(x)) = x.$$

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1)

1) 1997

2)

2) 4

3)

3) 30

4)

4) 23

5)

5) 27

6)

6) 6

7)

7) 1

8)

8) d

9)

9) 2

10)

10) [99, 101]

(OVER)

11)

11) d

12)

12) c

13)

13) 24

14)

14) 32

15)

15) b

16)

16) $\frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

17)

17) $\frac{1}{3}$

18)

18) 60

19)

19) b

20)

20) -3,0

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find a complex number z satisfying $(1 - i)z = 3 + 4i$. Write your answer in the form $a + bi$. 1) _____

2) Find the volume of a cube with edge length $(n + 1)$ if a cube with edge length n has volume 8. 2) _____

3) What is the value of x in the equation below?

$$\log_x(10) + \log_x(10^2) + \cdots + \log_x(10^{10}) = 110. \quad 3) \text{ _____}$$

(a) $\sqrt{10}$ (b) $e + 1$ (c) 10 (d) 20

4) How many ways can you make change for a twenty dollar bill with \$1, \$5, and \$10 bills? 4) _____

5) What is the area of the region bounded by the lines $y = 3 - \frac{4x}{9}$, $x = 3$, and the x - and y -axes? 5) _____

6) What integer is equal to the following sum?

$$\frac{111}{1 + 1 + 1} + \frac{222}{2 + 2 + 2} + \cdots + \frac{999}{9 + 9 + 9} \quad 6) \text{ _____}$$

7) If $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$ for $k \neq 0$, what is the value of r ?

(a) $-\log_2(5)$ (b) $\log_2(2)$ (c) $\log_{10}(5)$ (d) $\log_2(5)$ 7) _____

8) Over its domain, the following trigonometric expression equals what integer?

$$\cos^2(x)(1 + \tan^2(x)) \quad 8) \text{ _____}$$

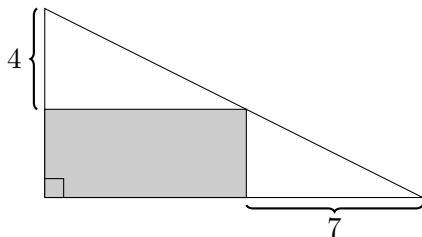
9) How many solutions are there to the equation $|2x^2 - x - 1| = x$?

(a) 1 (b) 2 (c) 3 (d) 4 9) _____

10) If $\varphi = \frac{1 + \sqrt{5}}{2}$ is the golden ratio, what integer is equal to $\varphi^2 - \frac{1}{\varphi}$? 10) _____

(OVER)

11) Find the area of the shaded rectangle below.



11) _____

12) Find $\sec(\theta)$ if $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{13}}{5}$.

12) _____

13) The first and last terms of an arithmetic sequence are 6 and 22. The sum of the sequence is 112. How many terms are in the sequence?

13) _____

14) Find all real x in the interval $[0, 2\pi)$ satisfying the equation $\sin^2(x) + \sin(x) = 0$.

14) _____

15) The following system of equations has a unique solution (a, b) . Find the value of $\frac{1}{a} + \frac{1}{b}$.

$$\begin{cases} \frac{1}{x} + \frac{3}{y} = 7 \\ \frac{4}{x} - \frac{2}{y} = 1 \end{cases}$$

15) _____

- (a) $\frac{22}{7}$ (b) $\frac{57}{7}$ (c) $\frac{62}{5}$ (d) $\frac{6}{5}$

16) What is the period of the function $f(x) = \cos(\sin(x))$?

16) _____

17) In $\triangle ABC$, $\cos(2A - B) + \sin(A + B) = 2$, and the length of $\overline{AB} = 4$. What is the length of $\overline{BC}$?

17) _____

- (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) $2\sqrt{3}$

18) Find the exact value of $\cos 1^\circ + \cos 2^\circ + \cdots + \cos 179^\circ + \cos 180^\circ$.

18) _____

- (a) -1 (b) 0 (c) 1 (d) 2

19) How many real solutions are there to the equation $x^x = x$?

19) _____

20) If $f(1) = f(2) = 1$ and $f(n) = f(n-1) - f(n-2) + 2$ for $n \geq 3$, then what is the value of $f(2024)$?

20) _____

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1)

1) $-\frac{1}{2} + \frac{7}{2}i$

2)

2) 27

3)

3) a

4)

4) 9

5)

5) 7

6)

6) 333

7)

7) d

8)

8) 1

9)

9) b

10)

10) 2

(OVER)

11)

11) 28

12)

12) 25

13)

13) 8

14)

14) $0, \pi, 3\pi/2$

15)

15) a

16)

16) π

17)

17) C

18)

18) a

19)

19) 2

20)

20) 1

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) A fair 10-sided die is rolled. What is the probability a prime number was rolled?

1) _____

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{3}{10}$ (d) $\frac{3}{5}$

- 2) What integer is equal to $\sqrt[3]{98 \cdot 56 \cdot 32}$?

2) _____

- 3) If the radius of a sphere is halved, what is the ratio of the new volume to the old?

3) _____

(a) 1:8 (b) 1:4 (c) 1:6 (d) 1:2

- 4) What integer is equal to $(i - 1)^8$?

4) _____

- 5) An isosceles has two sides of length 10 and one of length 12. What is its area?

5) _____

(a) 36 (b) 48 (c) 64 (d) 72

- 6) What integer is equal to the following expression?

6) _____

$$\log_3(7) \cdot \log_5(9) \cdot \log_7(11) \cdot \log_9(13) \cdots \log_{21}(25) \cdot \log_{23}(27)$$

- 7) Find all solutions to the following equation.

7) _____

$$\frac{(x!)^2}{(x-1)!(x+1)!} = \frac{11}{12}$$

- 8) What is the maximum value of the function $f(x) = \sqrt{3}\sin(x) + \cos(x)$?

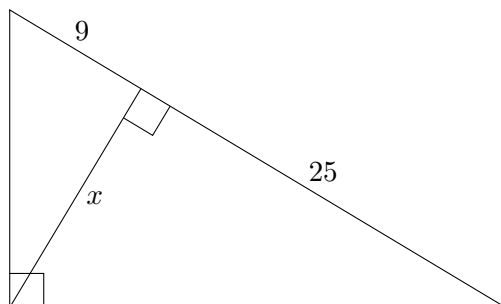
8) _____

- 9) If $f(x) = ax^4 - bx^2 + x + 5$ and $f(-3) = 2$, then what is $f(3)$?

9) _____

- 10) Find the length x in the diagram below.

10) _____



(OVER)

11) If $\sin x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, find $\cos(3x)$. 11) _____

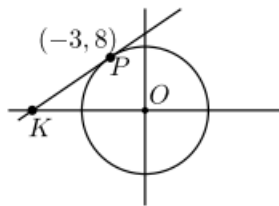
12) Evaluate $1 - 2 + 3 - 4 + \cdots - 98 + 99$. 12) _____
 (a) -50 (b) -49 (c) 0 (d) 50

13) A circle has a radius of $\log(a^2)$ and a circumference of $\log(b^4)$. What is the value of $\log_a(b)$? 13) _____

14) For exactly two real values of m , m_1 and m_2 , the line $y = mx + 3$ intersects the parabola $y = x^2 + 2x + 7$ at exactly one point. What integer is equal to $m_1^2 + m_2^2$? 14) _____

15) Suppose that in base b , $(122)_b = 50$ (where 50 is in base 10). What is b ? 15) _____

16) In the figure shown, a line is tangent to the circle centered at the origin. The point of tangency is $(-3, 8)$. The line intersects the x -axis at $x = k$. Find k .



16) _____

(a) $-\frac{73}{3}$ (b) $-\frac{8}{3}$ (c) $-\frac{64}{3}$ (d) $-\frac{16}{3}$

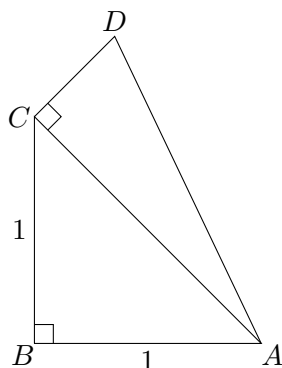
17) If $\log_2(x) = \log_y(16)$ and $xy = 64$, what is the value of $\left(\log_2\left(\frac{x}{y}\right)\right)^2$? 17) _____

18) Which of the following is equal to $\sin 80^\circ \cos 65^\circ - \cos 80^\circ \sin 65^\circ$?

(a) $\frac{\sqrt{1-\sqrt{3}}}{2}$ (b) $\frac{\sqrt{2+\sqrt{3}}}{2}$ (c) $\frac{\sqrt{2-\sqrt{3}}}{2}$ (d) $\frac{\sqrt{1+\sqrt{3}}}{2}$ 18) _____

19) A deck of cards has only red cards and black cards. The probability of a randomly chosen card being red is $\frac{1}{3}$. When 4 black cards are added to the deck, the probability of choosing red becomes $\frac{1}{4}$. How many cards were in the deck originally? 19) _____

20) Right triangle $\triangle ACD$ is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle $\triangle ABC$ with leg length 1. If the two triangles have the same perimeter, what is the value of $\sin(2\angle BAD)$?



20) _____

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1)

1) b

2)

2) 56

3)

3) a

4)

4) 16

5)

5) b

6)

6) 6

7)

7) 11

8)

8) 2

9)

9) 8

10)

10) 15

(OVER)

11)

11) $\frac{-44}{125}$

12)

12) d

13)

13) π

14)

14) 40

15)

15) 6

16)

16) a

17)

17) 20

18)

18) c

19)

19) 12

20)

20) $\frac{7}{9}$