LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University - 2010 Junior Examination
(Section I)

NAME: _____   Address: _____

SCHOOL: _____   City/ZIP: _____

                                            Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations.
 Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Express $\dfrac{(0.01)^3}{(0.5)^2}$ as a fraction in lowest terms.   1)_____

2) The average of three numbers is 21. If two of the numbers are 4   2)_____
   and 18, what is the third number?

3) Reduce $\dfrac{x^3 + 27}{x^2 - 2x - 15} \cdot \dfrac{2x - 10}{x^2 - 3x + 9}$ to lowest terms.   3)_____

4) If the perimeter of a square is $\dfrac{1}{3}$ the area of the same square, then   4)_____

   what is the length of the side of the square?

5) Find the vertex of the parabola $y = 2x^2 + 6x + 3$.   5)_____

6) The sum of the real solutions to $x^2 - 2|x| - 15 = 0$ is equal to: 6)_____
   (a) 0   **(b)** -2   **(c)** 2   **(d)** 8

7) Find all real solutions to $\ln(x + 1) - \ln x - 5 = 0$.   7)_____

8) Matt, Valerie, and Roy have a total of $5.50 in nickels. Matt has   8)_____
   3 times as many nickels as Valerie and 2 times as many nickels as
   Roy. How many nickels does Roy have?

9) The number of zeros of $f(x) = \left(\dfrac{1}{2}\right)^x - \sin x$ on the interval   9)_____

   $[0, 2\pi]$ is
      (a) 1   **(b)** 2   **(c)** 3   **(d)** 4

10) If $|x + 2| + |x + 1| > k$ for all real numbers x,   then what is the   10)_____
    range of values for k ?

(OVER)

11) If $\log_a 10 = B^x$ and $B^x = 2$, then a equals ?

12) Suppose $f(x) = x^2 + bx + c$ has exactly one real x-intercept and this x-intercept is also the x-intercept of the line $y = 3x + 4$, what is the value of b ?

12) $b =$ _____

13) There is a square $\square ABCD$. M and N are midpoints of $\overline{BC}$ and $\overline{CD}$, respectively. A point is selected at random inside the square. What is the probability that this point lies within $\triangle MCN$?

13)_____

14) If $f(x) = 4x^2 - kx - 8$ is a monotone function on $[5, 8]$, then what is the range of k ?

14)_____

15) Suppose $f(x) = 1 - \sqrt{x - 1}$, where $x \geq 1$, then $f^{-1}(x)$ is

15)_____

   (a) $(x - 1)^2 + 1,\ x \in \mathbb{R}$ **(b)** $(x - 1)^2 - 1,\ x \in \mathbb{R}$

   (c) $(x - 1)^2 + 1,\ x \leq 1$ **(d)** $(x - 1)^2 - 1,\ x \leq 1$

16) What is the largest perfect square divisor of 8,432 ?

16)_____

17) If a complex number z satisfies $|z + i| + |z - i| = 2$, then the smallest value attained by $|z + i + 1|$ is :

17)_____

   (a) 2   **(b)** 1   **(c)** $\sqrt{5}$ **(d)** 3

18) What is the value of $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$ if $\tan\theta = \sqrt{2}$?

18)_____

19) If $a^{2x} = \sqrt{2} - 1$, then the value of $\dfrac{a^{3x} + a^{-3x}}{a^x + a^{-x}}$ is :

19)_____

   (a) $2\sqrt{2} - 1$ **(b)** $2 - 2\sqrt{2}$

   (c) $2\sqrt{2} + 1$ **(d)** $\sqrt{2} + 1$

20) If x and y satisfy $\begin{cases} x - y \geq 0 \\ x + y \leq 1 \\ \quad y \geq -1 \end{cases}$ then what is the smallest value attained by $2x + y$?

20)_____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the midpoint of the line segment with endpoints (-6, 0) and (4, 3)?

1)_____

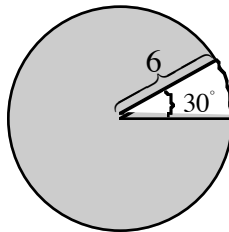2) What is the probability of flipping a fair coin 3 times and obtaining exactly 2 tails and 1 head?

2)_____

3) If $\log_2 3 = a$ and $\log_2 5 = b$, then $\log_2 \dfrac{9}{5}$ can be expressed as

   (a) $a^2 - b$ (b) $\dfrac{2a}{b}$ (c) $\dfrac{a^2}{b}$ (d) $2a - b$

3)_____

4) Given a function $f(x) = ax^4 - bx^2$, if $f(-1) = 1$, then what is $f(1)$?

4)_____

5) Find all real numbers x such that $\dfrac{4}{x + 2} > \dfrac{3}{x - 1}$. Express your answer using interval notation.

5)_____

6) Find the area of the **shaded** region.

6)_____

7) Find all real solutions to the following system:

$$\ln xy - 2\ln y = \ln 8$$

$$\ln \frac{x}{y} + 2\ln y = \ln 2$$

7)_____

8) Find all real solutions to $x^3 + 4x^2 - 8x - 32 = 0$.

8)_____

9) If $(x + 2)(x + k)^2 = x^3 + 16x^2 + 77x + 98$, then k equals ?

9)_$k =$_____

10) What is the domain of $y = \dfrac{\sqrt{x^2 + 2x - 8}}{x^2 - x - 12}$?

10)_____

(OVER)

11) Find all real solutions of $\sin^2 x + \dfrac{\sin x}{2} - \dfrac{1}{2} = 0$ on $[0, 2\pi]$.   11)_____

12) What is the coefficient of x^3 in the expression $(x^2 + 1)(x - 2)^7$?   12)_____

13) If a sequence $\{a_n\}_{n \geq 1}$ with $a_1 = 1$ has a partial sum s_n, and   13)_____

a point (a_n, a_{n+1}) is on the line $x - y + 1 = 0$, then

$\dfrac{1}{s_1} + \dfrac{1}{s_2} + \ldots + \dfrac{1}{s_n}$ is equal to :

(a) $\dfrac{n(n + 1)}{2}$ (b) $\dfrac{2}{n(n + 1)}$ (c) $\dfrac{2n}{n + 1}$ (d) $\dfrac{n}{2(n + 1)}$

14) Find all real values x such that $2^{\log_4 x} = 64$.   14)_____

15) Given $f(n) = \begin{cases} 1, & n = 0 \\ n \cdot f(n - 1), & n \text{ a positive integer} \end{cases}$, then $f(6) = $?   15)_____

16) If $x, y > 0$ and $\ln 2^x + \ln 8^y = \ln 2$, then the smallest value   16)_____

attained by $\dfrac{1}{x} + \dfrac{1}{3y}$ is

(a) 2   (b) $2\sqrt{2}$ (c) 4   (d) $\sqrt{3}$

17) Define $P = \{x \mid x^2 - 4x - 5 < 0\}$ and $Q = \{x \mid |x| - a \geq 0\}$.   17)_____

If $P \cap Q = \varnothing$, then a satisfies

(a) $a > 2$ (b) $a \geq 5$ (c) $-1 < a < 5$ (d) $a > 1$

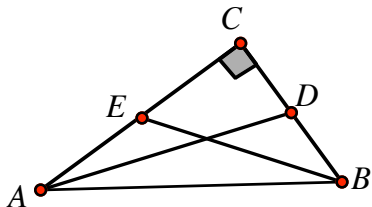**18)** If $|x| \leq \dfrac{\pi}{4}$, then what is the minimum value of   18)_____

$f(x) = \cos^2 x + \sin x$?

19) If a straight line $x + y = 1$ does **not** intersect the circle   19)_____

$x^2 + y^2 - 2ay = 0$, where $a > 0$, then a is a member of

which of the following intervals?

(a) $\left(0, \sqrt{2} - 1\right)$ (b) $\left(\sqrt{2} - 1, \sqrt{2} + 1\right)$

(c) $\left(-\sqrt{2} - 1, \sqrt{2} - 1\right)$ (d) $\left(0, \sqrt{2} + 1\right)$

20) In triangle ABC given below, $\angle ACB = 90°$, D and E 20)_____

are midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. If $\overline{BE} = 4$ and

$\overline{AD} = 7$, then what is $\overline{AB}$?

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University - 2010 Senior Examination
(Section I)

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                                             Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations.
Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the equation of the line, in slope-intercept form, that   1)_____
   is perpendicular to $2x - 3y = 8$ and passes through $(1, 4)$?

2) $\left| 3 - \pi \right| =$ 2)_____
   (a) $3 - \pi$ **(b)** $\pi - 3$ **(c)** $-3 - \pi$ **(d)** $3 + \pi$

3) Suppose the area of a circle is to be doubled. This means the radius   3)_____
   of the original circle must be increased by a factor of

   (a) 2 **(b)** $\dfrac{1}{2}$ **(c)** $\sqrt{2}$ **(d)** $\dfrac{\sqrt{2}}{2}$

4) (a, b) and (b, a) are points in the Cartesian plane such that $a \neq b$.   4)_____
   If the distance between (a, b) and (b, a) is 5 units, then what
   is $\left| a - b \right|$?

5) Find all real numbers x such that the slope of the line segment   5)_____
   passing through the points $\left(x^2, 7\right)$ and $(-2, 3)$ is $\dfrac{1}{8}$.

6) If a straight line $x - y + 6 = 0$ is perpendicular to a straight line   6) $a =$ _____
   $(a + 2)x - y + 1 = 0$, then what is the value of a ?

7) If a set $A = \left\{ x \mid ax^2 - 3x + 2 = 0 \right\}$ has at most one element, then   7)_____
   what is the range of a ?

8) Find all real solutions to $e^{\log_8 x} = 50$.   8)_____

9) If $9a^2 + ka + \dfrac{1}{9}$ is a complete square, then k equals :   9)_____
   (a) 2 **(b)** -2 **(c)** ± 2 **(d)** ± 1

10) What is the remainder when $x^4 + 20x^3 - x + 5$ is divided   10)_____
    by $x + 5$?

(OVER)

11) In the expression $\left(\dfrac{1}{x} + x^2\right)^6$, what is the coefficient of x^3 ?

11)_____

12) Given a function $f(x) = x^3 + \sin x + 1$, $x \in \mathbb{R}$. If $f(a) = 2$, then what is $f(-a)$?
(a) 3   (b) 0   (c) -1   (d) -2

12)_____

13) If $f(\log_2 x) = 2^x$, then $f(3)$ equals
(a) 128   (b) 256   (c) 512   (d) 8

13)_____

14) Define a function $f(x) = \begin{cases} x + 2, & x \le -1 \\ x^2, & 1 < x < 2 \\ 2x, & x \ge 2 \end{cases}$. If $f(x) = 3$, then what is the value of x ?

14)_____

15) Find all real solutions to $\cos(\arcsin x) = x + 1$.

15)_____

16) If $\left(x^2 + y^2\right)\left(x^2 + y^2 - 2\right) + 1 = 0$, then what is the value of $x^2 + y^2$?

16)_____

17) What is the greatest common divisor of 2016 and 384?

17)_____

18) If $a = \sqrt{2} - 2$, what is the value of
$$\dfrac{4}{a^2 - 4} + \dfrac{2}{a + 2} - \dfrac{1}{a - 2} \;?$$

18)_____

19) What is the minimum value attained by
$$f(x) = \log_2\left(x + \dfrac{1}{x - 1} + 5\right) \;?$$

19)_____

20) The straight line $y = kx + b$ passes through a point $(1, 4)$, intersects the x-axis at A, and the y-axis at B, as shown in the figure below. O is the origin. If $\triangle AOB$ has minimal area, then
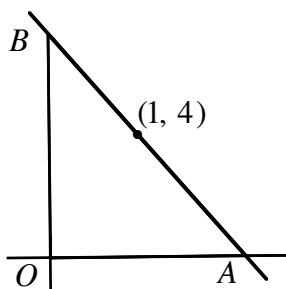
(a) $k = -4, b = 8$
(b) $k = -4, b = 4$
(c) $k = -2, b = 4$
(d) $k = -2, b = 2$

20)_____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) If $ab = 2$ and $c = 18a$, then what is bc? 1)_____

2) If the sum of 2 consecutive integers is 113, what is the larger of the two integers? 2)_____

3) If the area of an equilateral triangle is $16\sqrt{3}$ square units, then what is the length of a side of the triangle? 3)_____

4) If a substance loses half of its mass every 4 years, how much of a 100g sample of the substance will be left after 28 years? 4)_____g____

5) If $f(x) = \begin{cases} e^x, & x \le 0 \\ \ln x, & x > 0 \end{cases}$, what is $f\left(f\left(\dfrac{1}{2}\right)\right)$? 5)_____

6) Express $0.\overline{123}$ as a common fraction in lowest terms. 6)_____

7) Write $\dfrac{2 - 4i}{3 + 7i}$ in the form $a + bi$. 7)_____

8) If $y = kx$ and $k \ne 0$, then 8)_____
   (a) y is directly proportional to x.
   (b) y is inversely proportional to x
   (c) there is not enough information provided to determine if y is directly or inversely proportional to x.

9) During a sale, a store reduces the price of an item by 20%. By what percentage must the sale price be increased in order to obtain the original price of the item? 9)_____

10) Which of the following functions is equivalent to $y = x$? 10)_____
   (a) $y = \left(\sqrt{x}\right)^2$
   (b) $y = \sqrt[3]{x^3}$
   (c) $y = \sqrt{x^2}$
   (d) $y = \dfrac{x^2}{x}$
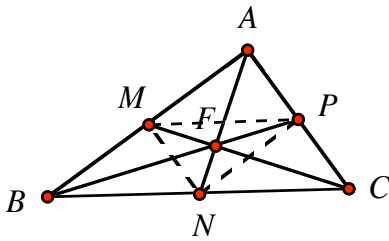
(OVER)

11) Find the domain of $f(x) = \dfrac{3x}{\sqrt{1-x}} + \ln(3x+1)$.

11)_____

12) How many subsets are there for the set $A = \{1, 3, 4, 5\}$?

12)_____

13) If $f(x) = (m-1)x^2 + (m-2)x + m^2 - 7m + 12$ is even,
then the value of m is
  (a) 1   **(b)** 2   **(c)** 3   **(d)** 4

13)_____

14) If the real solutions for x in the equation $\dfrac{m}{x+2} = 1$ are negative,
then what is the range of m ?

14)_____

15) If M, N, and P are midpoints of $\triangle ABC$, $\overline{BF}$ equals :

15)_____

  (a) $2 \cdot \overline{FP}$.
  (b) $3 \cdot \overline{FP}$
  (c) $\overline{FP}$
  (d) $\dfrac{1}{2} \cdot \overline{FP}$

16) Find all real x such that $\left(\dfrac{1}{3}\right)^{x^2 - 8} > 3^{-2x}$.

16)_____

17) Five different letters are chosen from $E\ Q\ U\ A\ T\ I\ O\ N$ and are
arranged in a sequence. How many arrangements can be made
with Q and U next to each other?

17)_____

18) If $\tan\theta = \sqrt{2}$, then what is the value
of $\sin^2\theta - \sin\theta\cos\theta + 2\cos^2\theta$?

18)_____

19) If $\sin\left(x + \dfrac{\pi}{4}\right) = -\dfrac{5}{13}$, then the value of $\sin 2x$ is equal to :

19)_____

  (a) $\dfrac{120}{169}$

  (b) $\dfrac{119}{169}$

  (c) $\dfrac{-120}{169}$

  (d) $\dfrac{-119}{169}$

20) If $a + b = 2$ where a and b are real, what is the smallest value
attained by $3^a + 3^b$?

20)_____