

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1993 Junior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

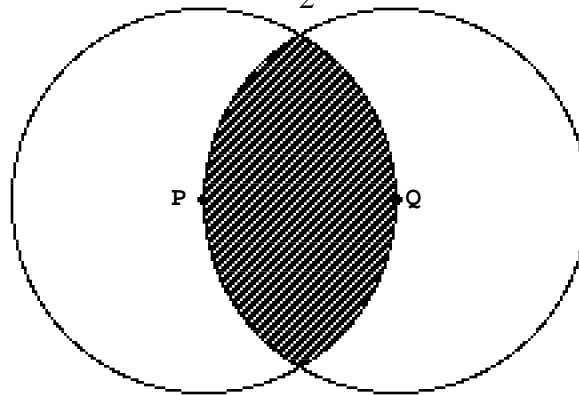
1) Solve for x : $7(x + 3) = 4(x + 5) - 47$

2) Determine the y -intercept of the line which passes through $(1,2)$ and has slope 4.

3) Find $\arcsin\left(\frac{5\pi}{4}\right)$.

4) State the amplitude and period of the function $f(x) = 2 \cos \frac{\pi x}{2}$.

5) In the figure shown, both the circle centered at P and the circle centered at Q have a radius of 1. What is the perimeter of the shaded region?



6) If $\log 2 = a$ and $\log 3 = b$, express $\log \frac{8}{9}$ in terms of a and b .

7) On the interval $[0, \pi]$, for what values of x is $\sin x + \cos x > 0$?

8) An automobile tire has diameter of 30 in. How many revolutions per minute will the wheel make when the automobile maintains a speed of 30 mi/hr? (Recall that there are 5280 feet per mile.)

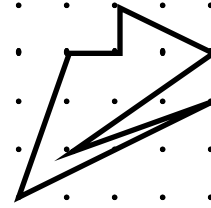
9) Determine the equation of the parabola which has vertex $(3,-4)$ and a horizontal axis of symmetry and which passes through the point $(1,-5)$.

10) Find all values of x satisfying $|3x + 2| = 5$.

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11) Express .000342 as the product of an integer power of 10 and a number between 1 and 10.

12) Give the area of the region enclosed by the figure shown, assuming that the distance between adjacent dots in a row or column is one unit.



13) Determine the value(s) of K for which the line $x = K$ is tangent to the circle $x^2 + y^2 - 4x + 2y + 1 = 0$.

14) Consider the points $A = (1,1)$ and $B = (4,7)$ in the plane. Determine x such that the point $C = (x,1)$ lies in the first quadrant and $AB = AC$.

15) State the domain of the function $f(x) = \frac{1}{\sqrt{3x+2}}$.

16) Solve for x : $\left(\frac{1}{4}\right)^x = 32$.

17) Suppose that 2 coins are tossed. Find the probability of obtaining at least one head.

18) Suppose f is a function satisfying $f(x)f(y) - f(xy) = x + y$ for all real numbers x and y . Determine the formula for $f(t)$.

19) Suppose $\csc \theta = \frac{17}{8}$ and $\frac{\pi}{2} < \theta < \pi$. What is the value of $\cos \theta$?

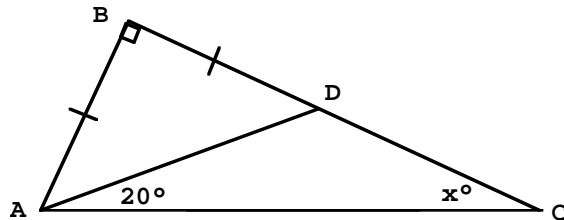
20) Starting from point A , a boat sails due south for 6 miles, the due east for 5 miles, and then due south for 4 miles. How far is the boat from A ?

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(Section II)

- 1) Determine all values of x satisfying $-5x \leq 30$.
- 2) State the value of $\tan 315^\circ$.
- 3) The daily payroll for a work crew is directly proportional to the number of workers, and a crew of 12 workers earns a payroll of \$540. What is the daily payroll for a crew of 15 workers?
- 4) Suppose $f(x) = x^2 + 3x - 4$. Find $f(2a + 3)$, and simplify your answer.
- 5) Solve for x : $\log_8(x - 6) + \log_8(x + 6) = 2$.

- 6) In the figure shown, ABD is a right isosceles triangle. Find x .



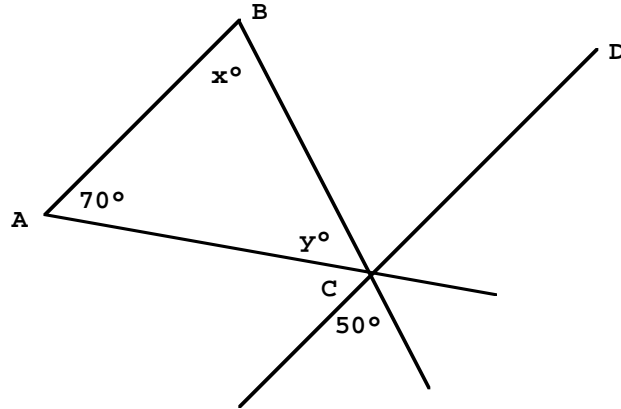
- 7) In a regular polygon, the measure of each interior angle is 162° . How many sides does the polygon have?
- 8) If the angle of elevation to the top of a tower from a distance 200 feet away from ground level is 60° , find the height of the tower.
- 9) A student's median score on three tests was 90. Her mean score was 92 and her range was 8. What were her high and low scores?
- 10) If, for all n , $2^n + 2^n + 2^n + 2^n = x(2^{n+1})$, then what is the value of x ?

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11) Determine all values of x in the interval $[0, 2\pi)$ such that $2\sin^2 x + \sin x = 1$.

12) The sum of two consecutive integers is 1 less than 3 times the smaller. Find the two integers.

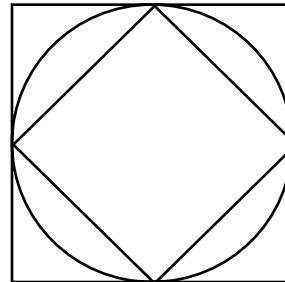
13) In the figure shown, $\overline{AB} \parallel \overline{CD}$.
 Find x and y .



14) Consider the lines with equations $5x - 2y = 10$ and $x - y = -1$. Find the coordinates of the point where these lines intersect.

15) Solve for B : $A = \frac{1}{2}h(b + B)$

16) In the figure shown, a small square is inscribed inside a circle, which is itself inscribed inside a large square. Find the ratio of the area of the large square to the area of the small square.



17) Suppose $f(x) = \frac{ax + b}{cx + d}$ where $bc - ad \neq 0$. Find the formula for the inverse function $f^{-1}(x)$.

18) Find a and b such that $(2 + i)(2 - i)(a + bi) = 10 - 4i$.

19) Find the polynomial p such that $(x - 2)p(x) = 2x^3 - x^2 - 5x - 2$ for all x .

20) A particle travels along a curve C in the xy -plane. At time t , the location of the particle is given by $x(t) = \cos^2 t$ and $y(t) = 2\sin t$. Find an equation of C in terms of x and y alone.