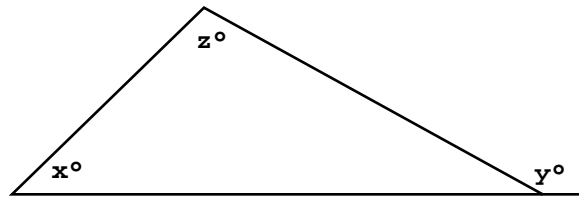


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Wilkes University – 1994 Junior Examination

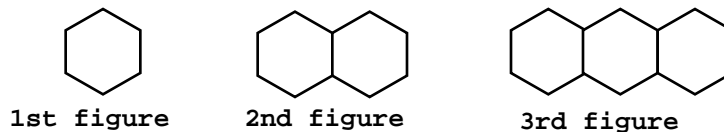
(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

- 1) Determine k such that $4(k - 3) + 5 = 7(3k - 1)$.
- 2) If a line l is perpendicular to the line $3x + 5y = 6$, what is the slope of l .
- 3) The line $ax + by + 6 = 0$ passes through the points $(-5, 2)$ and $(4, 1)$. Determine a and b .
- 4) Find all real numbers x satisfying $|2x - 3| < 7$.
- 5) If $x = 55$ and $y = 150$, find z .



- 6) A change box contains \$3.00 in dimes and nothing else. A certain number of dimes are removed and then replaced by an equal number of quarters, with the result that the box now contains \$4.20. How many dimes are removed?
- 7) Suppose that toothpicks are used to build the figure below. If the pattern were continued, how many toothpicks would be required to build the 100th figure?

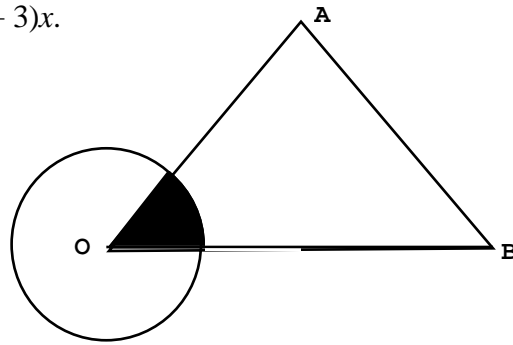


- 8) If $f(x) = x^2 - \frac{2}{x}$, find $f\left(\frac{2}{x}\right)$.
- 9) A ladder 5m. long is leaning against a building. The angle formed by the ladder and the ground is 60° . How far from the building is the foot of the ladder?
- 10) Solve for x : $\log_5(2x - 1) = 2$.

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11) Find all values x such that $x(2x - 5) = (x + 3)x$.

12) In triangle AOB , $AO = AB$, and the measure of $\angle A$ is 100° . The circle centered at O has a radius of 6. Find the area of the shaded sector.

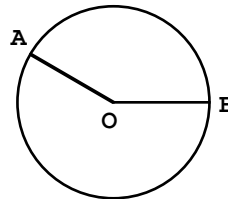


13) If $\sec \theta = -\frac{5}{3}$ and $\sin \theta < 0$, find $\tan \theta$.

14) Solve for x : $\frac{3x + 5}{2x - 3} = y$.

15) A rhombus has a side length of 4 inches and an angle of 60° . What is the area of the rhombus?

16) In the circle centered at O , the measure of $\angle AOB$ is 120° , and the length of the minor arc \widehat{AB} is 6π meters. Determine the radius of the circle.



17) Find the center and radius of the circle with equation $x^2 + y^2 - 6y = 16$.

18) Shannon spent \$50 on tapes, and then spent $\frac{2}{5}$ of her remaining money on books; after that, she used $\frac{1}{3}$ of the remaining amount to buy gifts. She was left with \$48. How much money did Shannon have initially?

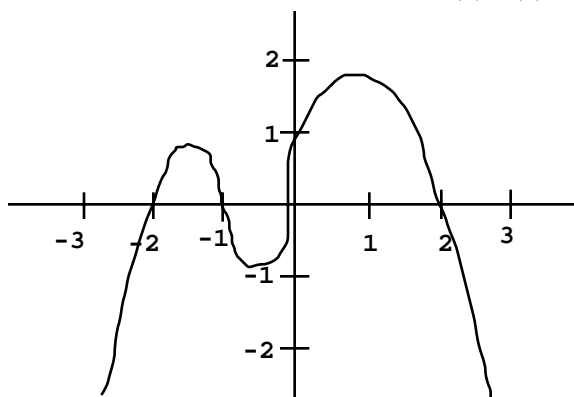
19) If the operation \square is defined $x \square y = 2x + 3y$ find $(4 \square 5) \square 6$.

20) Suppose $A(x)$ denotes the area of a triangle constructed as follows. A yardstick is cut into 2 pieces of lengths x and $3 - x$, and these pieces, together with a 1-foot stick, are used to form the triangle. Determine the domain and range of the function A . (Note the maximum possible area occurs when the triangle is isosceles)

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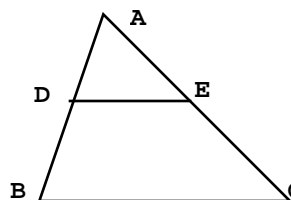
(Section II)

- 1) Solve for x : $\sqrt{x+3} = 2 + \sqrt{x-5}$
- 2) For a certain event, 812 tickets were sold totaling \$1912. If students paid \$2 per ticket and non-students paid \$3 per ticket, how many student tickets were sold?
- 3) In triangle ABC , $m\angle A = 50^\circ$, $m\angle C = 80^\circ$, $AC = 7x + 8$, and $BC = 38 - 3x$. Determine the value x .
- 4) Give the radian measure of a 330° angle.
- 5) Determine all roots of the equation $x^3 - 4x^2 + x + 6 = 0$.
- 6) The center of a circle lies in the second quadrant and is 1 unit from the y -axis and 2 units from the x -axis. If the circle is tangent to the y -axis, find the equation of the circle.
- 7) If the angle θ is acute and $\sin \theta = a$, express $\sin 2\theta$ in terms of a .
- 8) Solve for x : $4^{2x+1} = 8$.
- 9) Suppose f is a function with graph as shown. List all of the statements (a) - (e) which are true.



- (a) f is increasing on $[-1, 1]$
- (b) $f(0)f(2) > 0$
- (c) $f(0) + f(2) > 0$
- (d) $f(f(-1)) = 1$
- (e) f has only integer roots

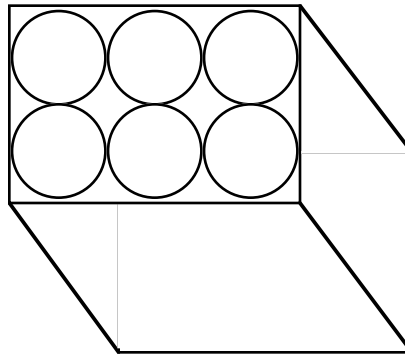
- 10) In triangle ABC , $AE = 6$, $CE = 10$, $AB = 14$, and $\overline{DE} \parallel \overline{BC}$. Find AD .



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- 11) Milton has $35\frac{1}{2}$ yd. of ribbon to make bows. Each bow requires $\frac{3}{8}$ yd. of ribbon.
(a) How many bows can be made?
(b) How much ribbon will be left over?
- 12) A parabola has a vertical axis and passes through the points $(-1,0)$, $(5,0)$, and $(1,8)$. Find the equation of the parabola.
- 13) Determine the period of the function $f(x) = \sin(6x - \pi)$.
- 14) Find all x in the interval $[0, 2\pi)$ such that $|\cos x| \leq \frac{\sqrt{2}}{2}$.
- 15) Determine A and B such that $\frac{2x - 2}{x^2 + 7x + 10} = \frac{A}{x + 2} + \frac{B}{x + 5}$.

- 16) A rectangular box is packed with six cylindrical soda cans, as shown. Find the ratio of total volume of the cans to the volume of the box.



- 17) Suppose each interior angle of a regular polygon measures 174° . How many sides does the polygon have?
- 18) A school has 1500 students. Each student takes 6 classes. Each teacher teaches 5 classes. Each class has 30 students and 1 teacher. How many teachers does the school have?
- 19) A multiple choice test consists of 5 questions, each with 3 possible answers. If a student guesses randomly on each question, what is the probability that she answers all questions correctly?
- 20) Suppose the fare for a taxi is \$2.50 for the first $\frac{1}{2}$ mile, plus 50 cents for each additional $\frac{1}{4}$ mile. Given that n denotes a positive integer, and $f(n)$ denotes the fare in dollars for an n -mile ride, give the formula for $f(n)$.