LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1995 Junior Examination
(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Solve for c: $s = \frac{1}{2}(a + b + c)$

2) The measure (in degrees) of the three angles of a triangle are $3x + 15$, $5x - 15$, and $2x + 30$. What is x?

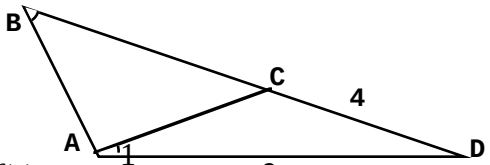
3) Suppose the radian measure of an angle θ is $\frac{9\pi}{4}$. What is the degree measure of θ?

4) Express the following in terms of $\sin\theta$ and $\cos\theta$, then simplify:
$$\frac{\sec\theta \csc\theta}{\tan\theta + \cot\theta}$$

5) Determine all values of x such that $|4x + 7| = 5$.

6) A farmer wishes to enclose a flat rectangular region with 90 ft. of fencing. The length of the rectangle is to be twice its width. What is the area of the enclosed region?

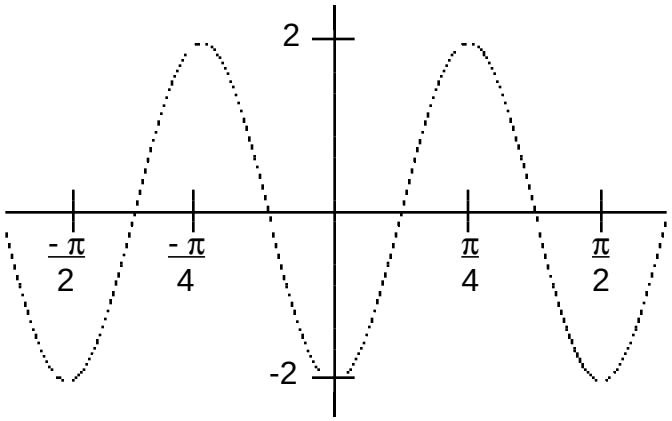
7) Given that $AD = 9$, $CD = 4$, and $m(\angle ABC) = m(\angle CAD)$, find BC.



8) Determine the domain of the function $f(x) = \frac{1}{\sqrt{3x - 4}}$

9) A certain test consists of 14 problems, with some problems worth 15 points each and the remaining problems worth 10 points each. If the test has a total of 200 points, how many 15-point problems are on the test?

10) Given that the curve below is the graph of a function having the form $f(t) = A\cos(Bt)$, determine A and B.



11) Determine the coordinates of the vertex of the parabola $x^2 - 8x - y = 3$.

12) Solve for x : $\log_{10}(2x + 1) - \log_{10}(x - 3) = 2\log_{10}3$

13) A box contains 3 red and 2 blue balls. Two balls are drawn at random without replacement. What is the probability of getting 1 red and 1 blue ball?

14) A wire stretches from the top of a vertical pole to a point on level ground 16 ft. from the base of the pole. If the wire makes an angle 60° with the ground, determine the length of the wire.

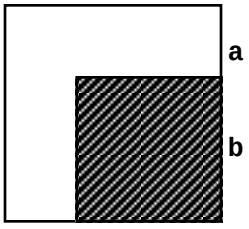
15) If $f(x) = 2x^3 - 6x^2 + x + k$, and $f(3) = -2$, find k..

16) From a group of boys and girls, 15 girls leave, after which there remain two boys for each girl. Then 45 boys leave, after which there remain five girls for each boy. How many girls were in the group at the beginning?

17) Suppose the length of each edge of a cube is increased by 50%. What is the resulting percent of increase in total surface area of the cube?

18) Solve for x: $3^{4x - 5} = 9$

19) In the square with side length $a + b$, the area of the shaded square is one third the total area. Determine the ratio $\frac{a}{b}$.



20) The area of a circle inscribed in an equilateral triangle is 48π. What is the perimeter of the triangle?

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(Section II)

1) Determine all real values of x such that $1 < \frac{4x - 1}{3} < 5$.

2) Determine k such that the line $kx + 3y = 0$ is perpendicular to the line $x - 2y = 0$.

3) If $\cos t = \frac{1}{2}$ and $0 \le t \le \pi$, find $\sin t$.

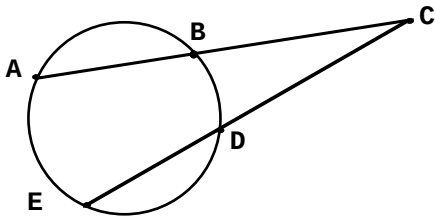
4) Find a such that $\log_a 16 = 2$.

5) Suppose m and n are integers such that $49 < m < 101$ and $19 < n < 51$. What is the largest possible value of $\frac{m + n}{m}$?

6) A 25 ft. ladder is placed against a vertical wall of a building, with the foot the ladder 7 ft. from the base of the building. If the top of the ladder slides 4 ft. download, how far does the foot of the ladder slide?

7) Suppose a line passes through the points (2,-9) and (-4,3). Determine the y –intercept of the line.

8) If $m(\overset{\frown}{AE}) = 90°$ and $m(\overset{\frown}{BD}) = 38°$, find $m(\angle C)$.



9) The negation of the statement “No dogs are purple” is:
(a) All dogs are purple.
(b) All dogs are not purple.
(c) Some dog is purple.
(d) Some dog is not purple.

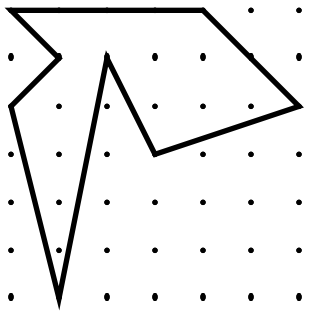
10) Determine x if $x^{\sqrt{\log_{10} x}} = 10^8$.

11) Determine a and b such that $\frac{a}{x+1} + \frac{b}{x-1} = \frac{x-2}{x^2-1}$.

12) For a gas under constant pressure, the volume is directly proportional to the temperature. Suppose at a temperature of 180° the gas occupies 100 cubic meters. What is the volume of the gas at a temperature of 150°

13) Find the coordinates of the point where the following lines intersect: $x + 2y = 1$ and $2x - y = 1$.

14) Determine the area of the region enclosed by the given figure, assuming that the distance between adjacent dots in a row or column is one unit.



15) Determine all values of x in the interval $[0,\pi]$ such that $2\cos^2 x - 1 = 0$.

16) If $f(2x - 1) = x^2$, determine $f(5)$.

17) Solve for x: $x + \sqrt{2x + 5} = 5$

18) Suppose x and y are related by the following equation:

$\ln y = 3 \ln x + \ln 5$

Express y as a function of x.

19) A talk show host will feature 3 women and 2 men as panelists, who will sit in a row of 5 chairs. How many different ways of seating the panelists are possible, if men and women are to be alternated?

20) A rectangle is inscribed in a circle of radius r. If x denotes the length of one side of the rectangle, express the area A in terms of x and r.