

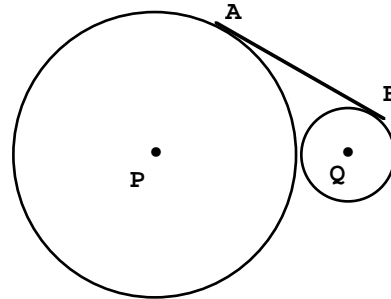
LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1996 Junior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

- 1) What is the slope of the line with equation $2x - 5y = 3$?
- 2) Express the fraction $\frac{11}{25}$ as a percent.
- 3) Solve for t : $q = r(s + t)$
- 4) Determine x so that the point $(x, 4)$ is equidistant from $(5, -2)$ and $(3, 4)$.
- 5) What is the minimum product of two numbers whose difference is 4?

- 6) The circles centered at P and Q have radii 6 and 2, respectively, and are tangent to each other. Find the length of their common external tangent \overline{AB} .



- 7) Determine all real values of x for which $|x - 3| = -(x - 3)$.
- 8) State the domain of the function $f(x) = \sqrt{9 - x^2}$.
- 9) A closed rectangular box of volume 10 cubic feet has a square base with side length x . If S denotes the total surface area of the box, express S as a function of x .
- 10) Suppose $f(x) = \begin{cases} 2x + 1, & \text{if } x \leq 0; \\ x^2 + 1, & \text{if } x > 0. \end{cases}$

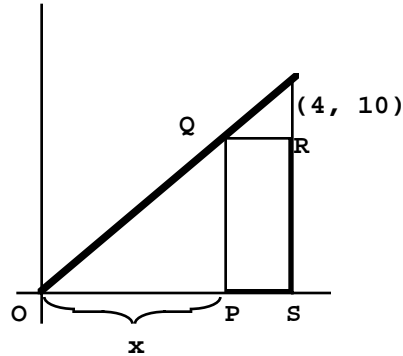
Which of the following statements are true?

- a) There is no value of x such that $f(x) = 17$.
- b) There is exactly one value of x such that $f(x) = 17$.
- c) There are exactly two value of x such that $f(x) = 17$.
- d) There are exactly three value of x such that $f(x) = 17$.

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11) If $\log_b 2 = 0.3$, then determine the value of $\log_b \left(\frac{b^5}{2} \right)$.

12) In the given diagram, let x denote the length of segment OP and A the area of rectangle $PQRS$. Express A as a function of x .



13) The number $\cos 460^\circ$ is:

- a) positive
- b) negative
- c) zero

14) Find the equation of the circle centered at $(2, 3)$ and tangent to the line $x = 5$.

15) Solve for x : $3 + 3^x = 12$

16) The current of a river is 2 mph. A boat travels to a point 8 miles upstream and back again in 3 hours. What is the speed of the boat in still water?

17) Evaluate: $\sin \frac{\pi}{3} \cos \pi + \cos \frac{\pi}{3} \sin \pi$

18) A ball is dropped from a height of 8 feet. On each bounce it rises to half its previous height. When the ball hits the ground for the seventh time, how far has it traveled?

19) Suppose A_1 denotes the area of a square inscribed in a semicircle and A_2 denotes the area of a square inscribed in the entire circle. Determine the ratio of A_1 to A_2 .

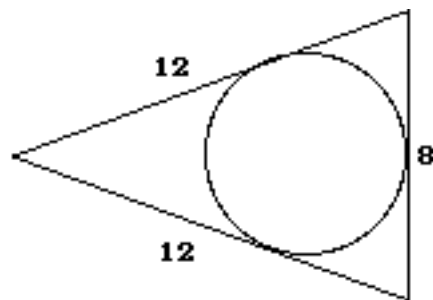
20) Among people who have a certain disease, the mortality rate is 30% for those who are not treated and the mortality rate is 5% for those treated. Suppose that among the people who have the disease, 80% have been treated. If a person is chosen randomly from those who have the disease, what is the probability that he will survive the disease?

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(Section II)

- 1) Solve for x : $\frac{5}{16}x + \frac{3}{8}x = 2 + \frac{1}{4}x$
- 2) If the degree measure of an angle θ is 560° , what is the radian measure of θ ?
- 3) Simplify: $\sec(\text{Arccos } \frac{1}{3})$
- 4) If $f(x) = \frac{x-4}{x+4}$, find $f(f(x))$ and simplify.
- 5) How much pure alcohol must be added to 7 liters of a 10% alcohol to obtain a mixture that is 30% alcohol?
- 6) Find all value of x which satisfy the inequality $33 - 12x < 4x + 97$.
- 7) Suppose the parabola $(y + 1)^2 = kx + 4$ passes through the point $(-6, 3)$. What is the value of k ?
- 8) Among the honor students at a certain high school, there are 20 students taking physics or chemistry. If 7 of these students are taking physics, and 3 are taking both chemistry and physics, how many are taking chemistry?

- 9) The given triangle has side lengths of 12, 12, and 8. Find the radius of the circle inscribed in the triangle.

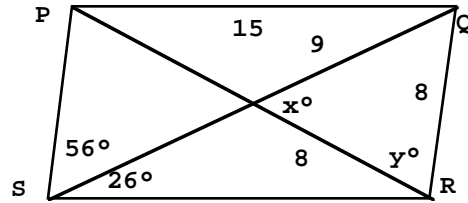


- 10) If $f(x) = \sqrt{2x + 3}$, find a formula for the inverse function $f^{-1}(x)$.

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- 11) Find the y -intercept of the perpendicular bisector of the line segment which joins the points $(2, 1)$ and $(-1, 5)$.
- 12) The side of a hill makes a 30° angle with the horizontal. If a person climbs 500 feet up the hillside, how far has she risen vertically?

- 13) Assuming that $PQRS$ is a parallelogram, determine the values of x and y .



- 14) From an ordinary deck of 52 playing cards, 2 cards are drawn without replacement. What is the probability that they are both red?

- 15) Find all ordered pairs (x, y) satisfying the following system of equations:

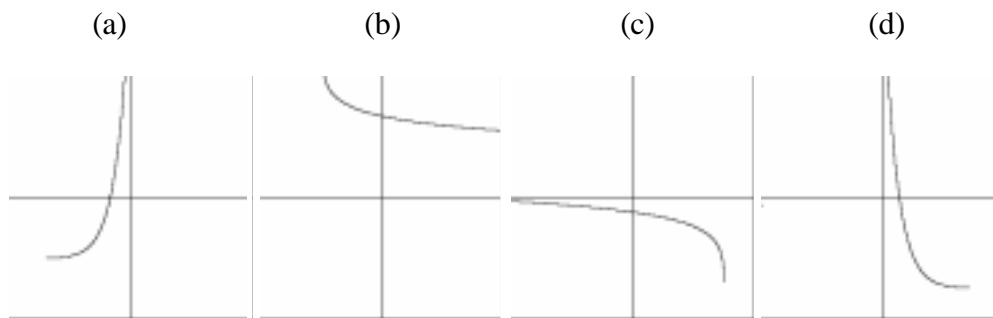
$$\begin{cases} xy = 8 \\ 3x + 2y = -16 \end{cases}$$

- 16) When s , is added to both the numerator and the denominator of a fraction $\frac{p}{q}$, the resulting fraction is $\frac{r}{s}$. Therefore, x equals

- a) $\frac{1}{r - s}$ c) $\frac{ps - qr}{r + s}$ e) $\frac{qr - ps}{r + s}$
 b) $\frac{ps - qr}{r - s}$ d) $\frac{qr - ps}{r - s}$

- 17) Solve for x : $8^x = 4^{x^2 - 1}$

- 18) If $b > 1$, then the function $f(x) = -\log_b x$ could be represented by which graph below?



- 19) The medians of a right triangle, drawn from the vertices of the acute angles, are 5 and $\sqrt{40}$. Find the length of the hypotenuse.

- 20) For each positive integer n , let $S_n = 1 - 2 + 3 - 4 + \dots + (-1)^{n-1}n$. Find the value of $S_{17} + S_{33} + S_{50}$.