

LUZERNE COUNTY MATHEMATICS CONTEST  
Luzerne County Council of Teachers of Mathematics  
Wilkes University – 1997 Junior Examination

(Section I)

**Directions:** Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) The number 8 is 2% of what number?

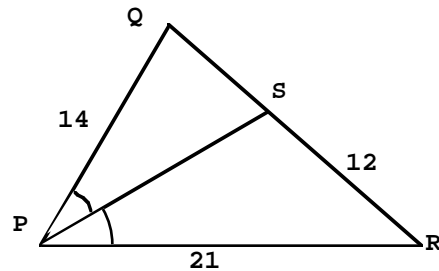
2) Solve for  $c$ :  $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$

3) What is the slope of any line perpendicular to the line  $2x + y = 3$ ?

4) Solve for  $x$ :  $|4 - 2x| = 3$

5) Determine  $c$  so that the point  $(-1, 3)$  lies on the line  $2x + cy = 5$ .

6) In the given triangle,  $m(\angle QPS) = m(\angle RPS)$ . If  $RS = 12$ ,  $PQ = 14$ , and  $PR = 21$ , determine  $QS$ .



7) Determine  $K$  so that the line with equation  $y = 4x + K$  is tangent to the parabola with equation  $y = 2x^2 - x + 1$ .

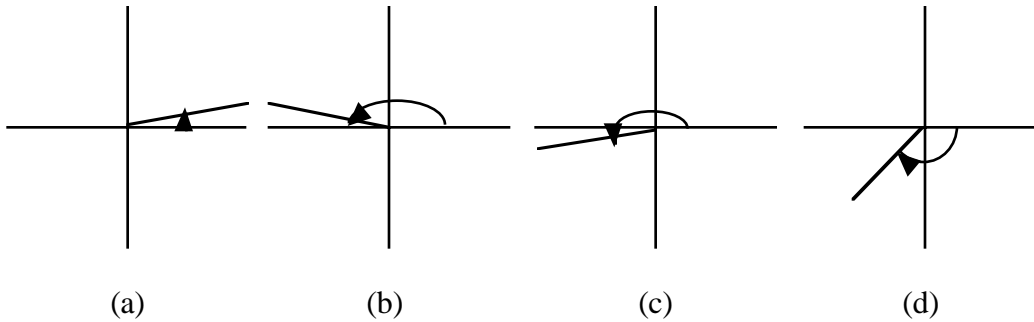
8) Ten points are marked on a circle. How many different triangles are there with all three vertices among the ten marked points?

9) Suppose that  $\tan x = \frac{2ab}{a^2 - b^2}$ , where  $0 < b < a$  and  $0 < x < \frac{\pi}{2}$ . What is  $\sin x$ ?

10) A square region is changed into a rectangular region by making it one foot wider and three times as long. If the area of the rectangular region is two square feet larger than the area of the original square region, what is the side length of the square before it was changed?

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11) Which of the following most closely approximates an angle of 3 radians in standard position?



12) Solve for  $x$ :  $\log_{10}(x + 2) - \log_{10}(x - 2) = 1$

13) The area of a circle inscribed in a regular hexagon is  $100\pi$ . Find the area of the hexagon.

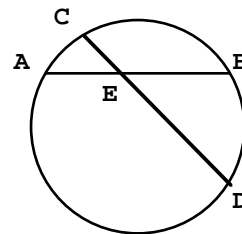
14) A radiator contains 8 qts of fluid, 40% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze in order to obtain a mixture that is 60% antifreeze?

15) Suppose that a wheel on a car has a radius 36cm. Find the angle (in radians) that the wheel turns while the car travels 5 meters.

16) Suppose  $Q = (7,5)$ ,  $R = (-2,2)$ , and  $T = (5,2)$ . Determine the coordinates of a point  $P$ , distinct from  $Q$ , such that  $\triangle PRT \cong \triangle QRT$ .

17) Among 50 freshman business students at a certain university 28 subscribe to Business Week and 32 students subscribe to Fortune. If 12 of the students subscribe to both magazines, then how many students subscribe to neither magazine?

18) Suppose  $CE = 9$ , and  $DE = 16$ , and  $AE = BE$ . Determine  $AE$ .



19) If  $f(x) = \frac{2x}{x - 3}$ , find the inverse function  $f^{-1}(x)$ .

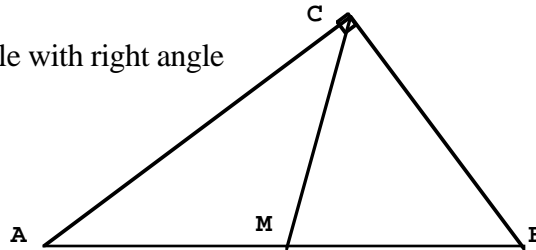
20) Find  $x$  and  $y$  so that both equations are satisfied:

$$\frac{4^x}{2^{x+y}} = 8 \text{ and } \frac{9^{x+y}}{3^{5y}} = 243$$

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- 1) Determine the  $y$ -intercept of the line with equation  $3x - 5y = 7$ .
- 2) Given a square with area  $A$  and perimeter  $P$ , express  $A$  as a function of  $P$ .
- 3) Find the equation of the parabola having  $(0, 0)$  at its vertex and the  $y$ -axis as its axis of symmetry and passing through point  $(2, 3)$ .
- 4) Determine all real values for  $x$  for which  $\frac{4}{x} < \frac{3}{5}$ .
- 5) Determine all non-zero values of  $a$  so that the equation  $ax^2 + 24x + 16 = 0$  has exactly one solution.

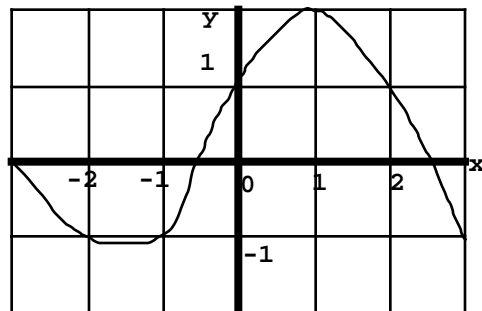
- 6) In the given figure,  $\triangle ABC$  is a right triangle with right angle at  $C$ , and  $\overline{CM}$  is a median of the triangle. If  $m(\angle CAM) = 34^\circ$ , determine  $m(\angle ACM)$ .



- 7) A factory offers 100 calculators to a retailer at a price of \$20 each. The price per calculator on the entire order will be reduced 5 cents for each additional calculator over 100. What number of calculators will produce the largest possible sales revenue for the factory?
- 8) Find the center and radius of the circle with equation:

$$3x^2 + 3y^2 + 12x + 12 = 18y$$

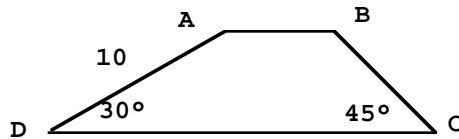
- 9) Below is the graph of the function  $y = f(x)$ . For which value(s) of  $x$  is  $f(x) = 1$ ?



- 10) Suppose the diameter of a wheel is 6 ft. How many revolutions of the wheel are required for a point on the rim to go one mile? (Recall 1 mile = 5280 ft.)

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- 11)  $\log_{10} 125$  equals: (a)  $100 \log_{10} 1.25$  (b)  $5 \log_{10} 3$   
(c)  $3 \log_{10} 25$  (d)  $3 - 3 \log_{10} 2$  (e)  $(\log_{10} 25)(\log_{10} 5)$
- 12) A person invests in a bond that yields 8% interest per year and also invests in a money market fund that yields 5% interest per year. Suppose the person obtains \$576 in interest for the first year. If the bond investment increases by \$100, then by how much must the money market investment be decreased in order to obtain the same total amount of interest for the next year?
- 13) Between 0 and  $2\pi$  inclusive, the function  $h(t) = \tan t$  has:  
(a) 3 roots and is undefined at 2 places.  
(b) 2 roots and is undefined at 3 places.  
(c) 2 roots and is undefined at 2 places.  
(d) 3 roots and is defined everywhere.  
(e) no roots and is undefined at 3 places.
- 14) Suppose that  $f$  is a function that  $f(1) = f(2) = f(3) = 1$ , and  $f(n+1) = \frac{f(n)f(n-1)+1}{f(n-2)}$  for  $n \geq 3$ . What is the value of  $f(6)$ ?
- 15) Solve for  $x$ :  $10^{x-1} = 100^{x+1}$
- 16) If  $\tan t = \frac{4}{3}$  and  $0 < t < \frac{\pi}{2}$ , what is  $\cos t$ ?
- 17) A local pizza shop offers eight different toppings. How many selections of two different toppings on a pizza are possible?
- 18) The sum of two numbers is 10; their product is 20. Find the sum of the reciprocals.
- 19) Suppose ABCD is a trapezoid with  $AD = 10$ ,  $CD = 20$ ,  $m(\angle ADC) = 30^\circ$ ,  $m(\angle BCD) = 45^\circ$ . Determine the area of the trapezoid ABCD.



- 20) Which expression in  $n$  gives the value of the following sum?

$$\sum_{k=1}^n (2k+1) = 3 + 5 + 7 + \dots + (2n+1)$$

- (a)  $n^2$  (b)  $n(n+1)$  (c)  $n(n+2)$   
(d)  $(n+1)^2$  (e)  $(n+1)(n+2)$