LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1998 Junior Examination
(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Find all real numbers x such that $\left| \frac{x - 5}{10} \right| \leq \frac{1}{5}$.

2) Express the infinite, repeating decimal $0.\overline{135} = 0.135135135...$ as a ratio of integers.

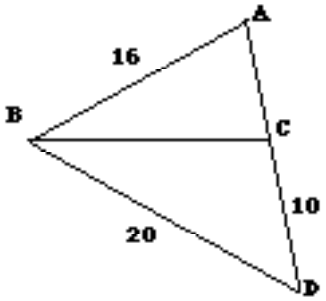
3) Find the center and radius of the circle whose equation is given as $3x^2 + 3y^2 + 12x + 12 = 18y$.

4) A student has test grades of 64 and 78. What grade must she get on a third test in order to have an average of 80?

5) Arrange the following numbers from smallest to largest.

$$20^{10},\ 20!/10!,\ 10^{20}$$

6) In the triangle shown, $m(\angle ABC) = m(\angle DBC)$. If $\overline{AB} = 16$, $\overline{BD} = 20$, and $\overline{CD} = 10$, determine $\overline{AC}$.



7) Find two positive real numbers x and y whose product is $\frac{1}{2}$ and the sum of their squares is 1.

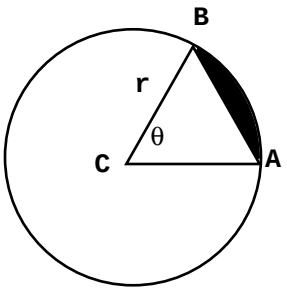
8) If $v \neq 0$, then simplify: $\left(\frac{2u^{-2}}{v^3}\right)^{-1} \left(\frac{4u^{-1}}{v^2}\right)^3$

9) Find all real numbers x such that $3^{x^2 - 15} = 9^x$.

10) How many ways are there to place 10 *identical* balls into 3 distinct urns labeled A, B, and C?

11) Find the equation of the line perpendicular to the line $y = -1.5x + 15$ which passes through the point (1, -2).

12) In the diagram, the shaded region is a sector of ABC of a circle cut off by chord $\overline{AB}$. Express the area of the shaded region in terms of the radius r and the central angle θ.



13) What is the domain and range of $f(x) = \log_{10}\sqrt{x^2 - 1}$?

14) The value of tan 300° is the same as

a) tan 60°
b) -tan 240°
c) -tan (-60°)
d) cot 30°
e) tan 240°

15) Solve for x: $1 - \frac{3}{x} = \frac{40}{x^2}$

16) Find the formula for $f^{-1}(x)$ if $f(x) = \frac{3}{4}x - 2$.

17) What is cos $2x$ if sin $x = \frac{5}{13}$ $(0 < x < \frac{\pi}{2})$?

18) A pharmacist has two solutions, the first containing 15% aspirin and the second 25% aspirin. How many ounces of each should be used to obtain 10 ounces of solution containing 20% aspirin?

19) Determine all real numbers b such that the equation $2x^3 + bx + 3 = 0$ has exactly one solution.

20) At a point on the ground 75 feet from the base of a flagpole, the angle of elevation of the top of the flagpole is 65°. Assuming sin 65° = .91, and cos 65°, find the height of the flagpole.

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(Section II)

1) Find the equation of the line which passes through the point (1, -3) and is parallel to the line $4x - 2y - 7 = 0$.

2) If the degree measure of an angle θ is 770°, what is the radian measure of θ?

3) Express in lowest terms: $\left(\frac{1}{x - y}\right)\left(\frac{y}{x} - \frac{x}{y}\right)$

4) Find three consecutive integers whose sum is 762.

5) A radiator contains 10 quarts of fluid, 30% of which is antifreeze. How much fluid should be drained with pure antifreeze in order that the new mixture will contain 40% antifreeze?

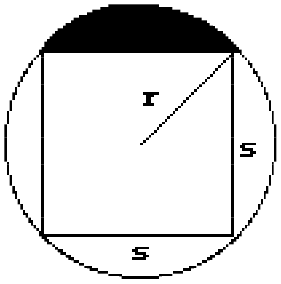
6) Find all values of x which satisfy the inequality

$$\frac{2x - 5}{x + 6} \leq 1.$$

7) Rewrite the equation $y = 5x^2 - 30x + 52$ in the form $y = a(x - h)^2 + k$, where a, h, and k are real numbers.

8) Find the domain and range of $f(x) = \sqrt{x - 7}$.

9) If a square is inscribed in a circle of radius r as shown, then find the area of the shaded region.



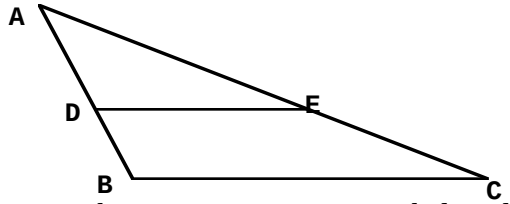
10) If $f(x) = \frac{1}{7}x + 3$, find a formula for the inverse function $f^{-1}(x)$.

11) Find $\csc\left(\text{Arcsin}\left(\frac{1}{16}\right)\right)$.

12) Find all real number solutions to the equation

$$\left| x^2 + 4x - 3 \right| = 2$$

13) In the figure shown, $\overline{DE} \parallel \overline{BC}$, $m(\overline{AD}) = 3$, $m(\overline{BD}) = 2$, and $m(\overline{DE}) = 4$. Find $m(\overline{BC})$.



14) Suppose that a bag contains three coins: a penny, a nickel, and a dime. Two coins will be drawn, without replacement, one at a time. What is the probability that the first coin will be a penny and the second coin will be a dime?

15) Solve for c: $\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

16) A ferris wheel makes 17 revolutions every 3 minutes. Then the angular speed of the ferris wheel, measured in radians per minute is

a) $\frac{17\pi}{3}$ b) $\frac{34}{3}$ c) $\frac{34\pi}{3}$

d) $\frac{3\pi}{34}$ e) $\frac{17}{3\pi}$

17) Find a polynomial $p(x)$ with *real* coefficients having degree 4 whose only roots are 4, $3 + i$, and $3 - i$.

18) If $A_n = 2^n - 2^{n-1}$, find $A_1 + A_2 + A_3 + \ldots + A_{10}$.

19) A company's profit is given (in thousands of dollars) by $P(x) = -50 + 25x - x^2$, where x is the number of units sold. What is the maximum possible profit?

20) If $\frac{3\pi}{2} < t < 2\pi$ and $\cos^2 t = \frac{16}{25}$, find $\csc t$.