

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1999 Junior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) If $a = \sqrt[3]{t}$ and $b = t^2$, then what is $\frac{b}{a^5}$?

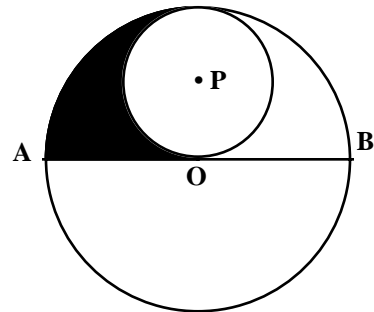
2) For what values of x is $x^2 + 6 < 5x$?

3) If 25 percent of a certain number is 36, what would 40 percent of the same number be?

4) If $a = \frac{b+x}{c+x}$, then what is the value of x in terms of a , b , and c ?

5) How many ways can two identical red chairs and 4 identical blue chairs be arranged in one row?

6) Circle O has a diameter AB of length 8. If the smaller circle is tangent to the diameter AB at the point O and is also tangent to the circle O , what is the area of the shaded region?



7) When $2x^3 + 3x^2 - 4x + k$ is divided by $x + 2$, the remainder is 3. What is the value of k ?

8) Find the quotient *and* remainder when $2x^2 - 3x + 6$ is divided by $x - 5$.

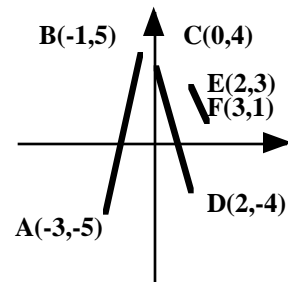
9) Find all real numbers x such that $2^{x^2-3} = 4^x$.

10) The original price P of a certain item is first discounted by 20%, and then 5% of the discount price is added for sales tax. If the final price, including sales tax is \$71.40, what was the original price?

LUZERNE COUNTY MATHEMATICS CONTEST
 Luzerne County Council of Teachers of Mathematics
 Wilkes University – 1999 Junior Examination

11) Find the equation of the line perpendicular to the line $9y - 3x = 2$ which passes through the point $(1, -2)$.

12) If the midpoints of \overline{AB} , \overline{CD} , and \overline{EF} , are connected, what is the area of the resulting triangle?



13) If $(x + y)^2 = (x - y)^2$, which must be true?

- a) $x = y$
- b) $x = -y$
- c) $x = 0$
- d) $y = 0$
- e) $x = 0$ or $y = 0$

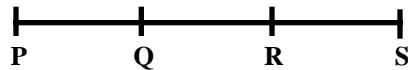
14) Simplify the expression $\tan\theta + \frac{1}{\tan\theta}$ into an expression involving *only* $\sin\theta$ and $\cos\theta$.

15) Solve for x : $\frac{5}{x+3} = \frac{1}{x} + \frac{1}{2x}$.

16) Find a formula for the inverse function $f^{-1}(x)$ if $f(x) = \frac{1}{3}x + 3$.

17) If $\cos 2A = \frac{7}{19}$, what is the value of $\frac{1}{\cos^2 A - \sin^2 A}$?

18) In the figure shown, the length of segment \overline{PS} is $2x + 12$ and the length of segment \overline{PQ} is $6x - 10$. If R is the midpoint of segment \overline{QS} , what is the length of segment \overline{PR} .



19) How many *integer* solutions are there to $|4x + 3| < 8$?

20) The side of a hill makes a 45° angle with the horizontal. If you climb 1000 feet up the hillside, how far will you rise vertically?

(Section II)

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1999 Junior Examination

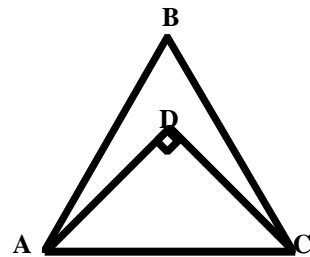
- 1) Find the equation of the line which passes through the point $(-1, -2)$ and is perpendicular to the line $2x - 3y + 4 = 0$.
- 2) If the point $(\sqrt{2}, -\sqrt{2})$ is given in rectangular coordinates, what is this point given in polar coordinates (r, θ) , where $0 \leq \theta \leq 2\pi$?
- 3) Express in lowest terms: $\left(\frac{x+y}{x+2y}\right) \div \left(\frac{x+y}{xy}\right)^2$.
- 4) A calculator cost the wholesaler \$28.00. What will the retailer sell it for if the wholesaler's markup is 20% of the wholesale cost and the retailer's markup is 30% of the retailer's?
- 5) If $\frac{2}{x}(x^2 + x) = \frac{1}{2}$, then what is $x + 1$?

- 6) Find all values of x which satisfy the inequality

$$\frac{2x - 5}{x + 6} \leq 1.$$

- 7) A new computer can perform x calculations in y seconds and an older computer can perform r calculations in s minutes. If these two calculations work simultaneously, how many calculations can be performed in t minutes?
- 8) Find the domain and range of $f(x) = \sqrt{x^2 - 1}$.

- 9) In the figure shown, $\triangle ABC$ is equilateral and $\triangle ADC$ is isosceles. If $AC = 1$ and $\overline{AD} \perp \overline{CD}$, then what is the distance from B to D ?



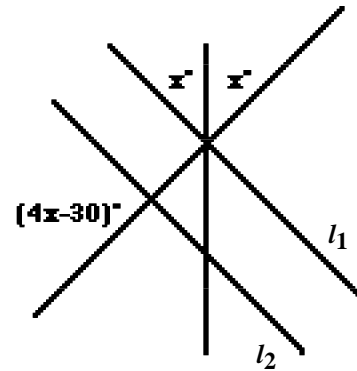
- 10) If $f(x) = 1 - 4x$, and $f^{-1}(x)$ is the inverse of $f(x)$, then what is the value of $f(-3)f^{-1}(-3)$?

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1999 Junior Examination

11) Find all real numbers x such that $\log_7(5x + 4) = 2$.

12) Evaluate the expression $\sec\left(-\frac{\pi}{4}\right)$.

13) In the figure shown, if $l_1 \parallel l_2$, then find the value of x .



14) A cube with edge length 4 is divided into 8 identical cubes. What is the difference between the combined surface area of the 8 smaller cubes and the surface area of the original cube?

15) What is the range of the function $f(x) = \frac{x^2}{x+1}$?

16) Express the complex number $(i + 1)^2(5 - 2i)$ in the form $a + bi$, where a and b are real numbers and $i^2 = -1$.

17) If $\cos x = 0.5$ and $-\frac{\pi}{2} < x < 0$, then find $\sin 2x$.

18) A committee of 3 women and 2 men is formed from a pool of 11 women and 7 men. How many different committees can be formed?

19) A square and a regular hexagon have the same perimeter. If the area of the square is 2.25 square units, what is the area of the hexagon?

20) The graph of the function shown in the figure could be a graph of

- a) $f(x) = (x - 2)^2$
- b) $f(x) = |x^2 - 4|$
- c) $f(x) = x^2 - 4$
- d) $f(x) = -|x^2 - 4|$
- e) $f(x) = 4 - x^2$

