LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2002 Junior Examination
(Section I)

NAME: _____ Address: _____

SCHOOL: _____ City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

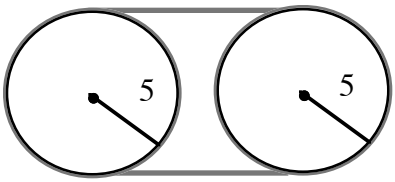
1) Find the equation, in slope-intercept form, of the line which passes through the point (1, 2) and is perpendicular to the line with equation $5x = 10y + 3$. 1) _____

2) How many distinct 7-letter strings can be generated from the letters in the word *ALABAMA?* 2) _____

3) Assume a daughter's, mother's and grandmother's combined age is 130 years. If the daughter age is one third that of the mother, and the grandmother's age is twice the mother's age, how old is the daughter? 3) _____ years

4) Find a number t, such that the line passing through ($t + 2$, 4) and (8, $-4t + 1$) has a slope of 3. 4) _____

5) If 2 gears, each of radius 5 in., are used in a chain drive system with a chain of length 60 in. , what is the distance between the centers of the gears? 5) _____ in.



6) The graph of the equation $xy^2 - x^3 - 7y^2 + 7x^2 = 0$ is that of 6) _____

A) a parabola
B) a hyperbola
C) 3 distinct lines
D) 2 distinct lines
E) none of the above

7) Find the length of the line segment connecting the points (5, 3) and (10, -9). 7) _____

8) How many real roots does $p(x) = x^6 - 6x^3 + x^2 - 2x + 11$ possess? 8) _____

9) Find all the real values of x such that $5x^2 \leq 100x - 500$, 9) _____

10) What is the smallest number of marbles that can be divided equally among 6 boys, then among 8 boys, then among 10 boys, and finally among 12 boys? 10) _____

(OVER)

11) Find A, B, and C such that

$$\frac{5x - 3}{(x - 1)(x - 2)^2} = \frac{A}{x - 1} + \frac{B}{x - 2} + \frac{C}{(x - 2)^2}$$

11) $A =$ ____ $B =$ ____ $C =$ ____

12) Find all real solutions to:

$$x^3 - 4x^2 + 5x = 2$$

12) _____

13) Find the area of the closed region bounded by the x-axis, the y-axis, and the line $y = -2x + 2$.

13) _____

14) Find the surface area of a sphere with radius 3m.

14) _____ m^2

15) Assume x is a real number in the domain of $f(x) = \frac{x^2 - 6x + 5}{x - 5}$. Then $f(x)$ also equals

A) $x - 1$
B) $(x - 1)(\sin^2 x + \cos^2 x)$
C) $(x - 1)(\sec^2 x - \tan^2 x)$
D) both A and B
E) both B and C

15) _____

16) What is the remainder when 3^{26} is divided by 7 ?

16) _____

17) Suppose $x + y = 2$ and $x^2 + y^2 = 20$. Find $x^3 + y^3$.

17) _____

18) A man has eight distinct shirts, four distinct pairs of pants and five distinct pairs of shoes. How many different outfits are possible?

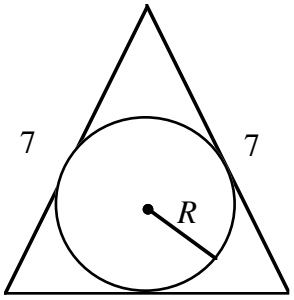
18) _____

19) Convert the rectangular coordinates $(1, \sqrt{3})$ to polar coordinates (r, θ) with $r > 0$ and $0 \le \theta \le 2\pi$.

19) _____

20) Solve for R , the radius of the circle inscribed in the triangle below.

20) _____



LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2002 Junior Examination
(Section II)

NAME: _____ Address: _____

SCHOOL: _____ City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Solve for x : $4^{5x} = 2sin(\frac{\pi}{6}) + 3cos(2\pi)$. 1) _____

2) Compute $log_2 (log_3 (9^8))$. 2) _____

3) Find the minimum value of the function $f(x) = 3x^2 - 6x + 10$. 3) _____

4) Find the constant term in the expansion of $(y + \frac{1}{2y})^6$. 4) _____

5) A man drives from home to work at a speed of 40 mph. The return trip is driven at a speed of 25 mph. What is the man's average speed on the round trip? 5) _____ mph

6) Find all real roots of $p(x) = x^3 - x^2 + 4x - 4$. 6) _____

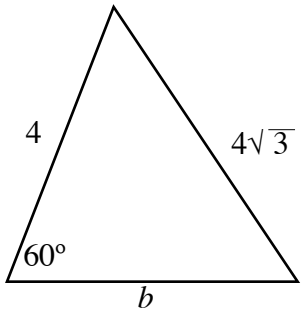
7) What is the probability that the sum of rolling two fair dice is a 3 or 11 ? 7) _____

8) The three numbers 3, 4, and 6 are : 8) _____

A) the sides of an acute triangle
B) the sides of an obtuse triangle
C) the sides of a right triangle
D) not the sides of any triangle

9) Express the following without radicals: $\sqrt{4 + 2\sqrt{3}} - \sqrt{4 - 2\sqrt{3}}$. 9) _____

10) Find the length b in the triangle below. 10) _____



(OVER)

11) Compute $\sin(\frac{5\pi}{12})$. 11) _____

12) Find the point (x, y) which lies on the line with equation $y = -3x + 7$ and on the line with equation $y = -5x + 21$. 12) $x =$ _____ $y =$ _____

13) Find the perimeter of the closed region bounded by the x-axis, the y-axis, the line $x = 3$, and the line $\frac{4}{3}x + y - 7 = 0$. 13) _____

14) A particle projected vertically upward reaches a velocity of $v(t) = 640 - 32t$ ft/sec at the end of t seconds. Find the time t (in seconds) when the particle reaches its maximum height. 14) $t =$ _____ sec.

15) The graph of the inequality $x^2 - 6x + y^2 - 10y \leq 66$ is 15) _____

A) a circle
B) a parabola
C) a closed circular disk
D) a hyperbola
E) none of the above

16) Which number best completes the following sequence? 16) _____
7, 19, 9, 18, 12, 18, 16, 19, __

A) 17 B) 21 C) 12 D) 19

17) Find the domain of the function 17) _____
$f(x) = \arcsin (2x + 3)$

18) Assume that three angles of a triangle are in arithmetic progression. Moreover, assume that the measure of the smallest angle (in radians) is one-half the measure of the largest angle. Find the measures (in radians) of the three angles. 18) _____

19) Suppose f and g are functions such that $g(4) = 2$, $f(4) = 5$, $f(2) = -3$ and $g(2) = 6$. Find $(f \circ g)(4)$. 19) _____

20) Find all real values of x that satisfy the following: 20) _____

$$\log_7 (x + 4) + \log_7 (x + 2) = 1$$