LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2003 Junior Examination
(Section I)

NAME: ______________________ Address: ______________________

SCHOOL: ______________________ City/ZIP: ______________________

Telephone: ______________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the equation of the line perpendicular to the line $y - 2x = 3x + 15$ that passes through the point (1, 4). Express your answer in slope-intercept form. 1) ______________

2) Suppose $f(x) = x^2 + kx + 3$. Find a value for k such that $f(2) = f(5)$. 2) k = ______________

3) For which value(s) of x does the graph of $f(x) = \dfrac{x^2 + 12x + 35}{(x+2)(x+5)}$ have a vertical asymptote? 3) ______________

4) Find the rectangular coordinates of the point whose polar coordinates are given as $(7, -\frac{2\pi}{3})$. 4) ______________

5) Find the sum of the following: $5 + \frac{5}{4} + \frac{5}{16} + \frac{5}{64} + \frac{5}{256} + \ldots$ 5) ______________

6) Find all real values of x satisfying $|x + 7| \geq 3$. 6) ______________

7) What is the probability of getting a sum of 6 or 8 when throwing 3 fair six-sided dice? 7) ______________

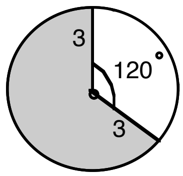
8) Find the value of $\sin\left(8 \arccos \frac{\sqrt{3}}{2}\right)$. 8) ______________

9) Suppose $f(x) = 2x^2 + 3$. Write $\dfrac{f(x+h) - f(x)}{h}$, in the form $ax + bh$, where $h \neq 0$. 9) ______________

10) Factor completely: $p(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$. 10) ______________

(OVER)

11) Find the area of the shaded region in the figure below. 11) ______________________



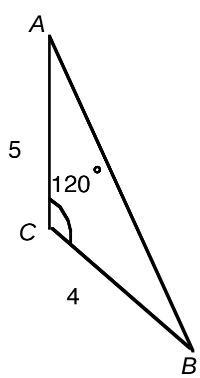
12) Find real numbers A and B such that 12) $A =$ __________ $B =$ __________

$$\frac{\sqrt{-6}\ \sqrt{-144}}{\sqrt{54}} = A + Bi$$

13) An urn contains 8 identical orange balls and 12 identical green balls. Three balls are drawn at random from the urn. What is the probability that at least 1 ball is **not** orange? 13) ______________________

14) Find the maximum value of the function $f(x) = -x^6 + 5x^3 - 3$. 14) ______________________

15) Find the length of $\overline{AB}$ in the triangle below . 15) ______________________



16) Express the constant $\log_2(e)\ \ln(4)$ without logarithms. 16) ______________________

17) Find the vertex of the parabola 17) ______________________

$$\sqrt{(3x + 3)^2 + y^2} = \sqrt{x^2 + (y + 3)^2}$$

18) Find the coefficient of $x^{56}y^2$ in the expansion of $(x^2 - 2y)^{30}$. 18) ______________________

19) Suppose $f(x) = 3x^5 - 30x^4 + 47x^3 + 8x^2 + x + 70$. Find $f(8)$. 19) ______________________

20) Assume $f(x) = \dfrac{(x - A)}{(Bx - C)}$. For what values of the constants A, B and C does $f(f(x)) = x$? 20) ______________________

i) $A = 2, B = -3, C = -1$

ii) $A = -4, B = 1, C = 1$

iii) $A = 0, B = 0, C = -1$

iv) *ii*) and *iii*)

v) *i*) and *ii*)

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2003 Junior Examination
(Section II)

NAME: ______________________________ Address: ______________________________

SCHOOL: ______________________________ City/ZIP: ______________________________

Telephone: ______________________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) For what value of k is the line $5x - ky = 7$ perpendicular to the line $3x + 8y = 4$? 1) ____________________

2) What is the largest prime number smaller than 100? 2) ____________________

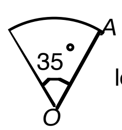
3) Suppose $f(x) = 3x(2 - x)$. Find all real values x such that $f(x) = x$. 3) ____________________

4) How many distinct diagonals does a pentagon have? 4) ____________________

5) The mean of eighteen numbers is 14. If three numbers are removed, the new mean is 17. What is the cube of the sum of the numbers that were removed? 5) ____________________

6) The diameter of a circle is 24 cm. By what amount must the radius be decreased in order to decrease the area by 80π cm^2 ? 6) ____________________ cm

7) Compute the length of the circular arc below: (in radians) 7) ____________________



length $\overline{OA}$ = 4

8) Assume we are provided with the following information of a pool of 1000 individuals - 8) ____________________

i) 600 are employed
ii) 800 are high-school graduates
iii) 500 are high-school graduates and are employed

What is the probability a person chosen from this pool is employed and **not** a high-school graduate?

9) Suppose there is a lottery in which 1 in 3 tickets wins a prize. If 3 tickets are purchased, what is the probability of winning a prize? 9) ____________________

10) Find the equation of the line, in slope-intercept form, that passes through the points (3, 6) and (8, -4). 10) ____________________

(OVER)

11) Suppose that a cube has edges of length s units. If each edge is increased by 2 units, express the increase in the cube's surface area in terms of s .

11) ______________________

12) Find all x such that: $4^{x^2} = 2^{8x - 6}$

12) ______________________

13) Find the domain of the function $f(x) = \dfrac{\sqrt{5x - 2}}{\sqrt{4x} - 3}$.

13) ______________________

14) Factor $4x^4 - 28x^2 + 24x$ into irreducible factors.

14) ______________________

15) Find all (x, y) satisfying the following system of equations.

$$4^x + 4^y = 80$$
$$2^x + 2^y = 12$$

15) ______________________

16) Find real numbers A and B such that

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{72} = A + Bi$$

16) A = ____________ B = _______

17) Find all real numbers θ, $0 \le \theta \le \frac{\pi}{2}$, which satisfy the equation $\dfrac{\sin 2\theta}{\sec 2\theta} = \dfrac{\sqrt{3}}{4}$.

17) ______________________

18) Express $\dfrac{(50!)^{51}}{(51!)^{50}} \bullet \dfrac{51^{48}}{48!}$ as a fraction in lowest terms.

18) ______________________

.

19) If $f(2) = I$, and , $f(3) = J$, $f(5) = K$, where I, J, and K are positive integers. Suppose $f(ab) = f(a)f(b)$ for all positive integers a and b. Find $f(360^n)$, where n is a positive integer, in terms of n, I, J, and K respectively.

19) ______________________

20) Find the area of the shaded region between the two concentric circles as shown in the figure below.

20) ______________________