LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University - 2004 Junior Examination
(Section I)

NAME: _____   Address: _____

SCHOOL: _____   City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the diameter of a circle with area 16π m^2?

1) _____ m

2) What is $\dfrac{3}{10}$ of $\dfrac{1}{2}$ of $\dfrac{57}{101}$?

2) _____

3) Find the length of a diagonal of a cube which has volume 8.

3) _____

4) What is the midpoint of the line segment joining points (1, 2) and (-4, 3) ?

4) _____

5) For which values of k does the equation $x^2 + 2k = -3x$ have exactly one (1) real solution?

5) $k =$ _____

6) Find the fraction, reduced to lowest terms, whose decimal representation is the repeating decimal $.\overline{36}$.

6) _____

7) A non-taxable $10 item is on sale for 60% off. A cashier makes an error and applies 2 successive reductions of 30% to the item. How much money does the customer lose due to the error?

7) $ _____

8) Find all solutions to $3 \tan \dfrac{x}{5} = \sqrt{3}$ on the interval $[4\pi, 6\pi]$.

8) $x =$ _____

9) Through how many radians does the hour hand on a clock move between 2:00 pm and 7:15 pm on the same day?

9) _____

10) Given that $\sin \theta = \dfrac{\sqrt{2}}{2}$, $\sin 2\theta$ is equal to

A) $\sqrt{2}$ C) -1

B) 1   D) this value cannot be uniquely determined

10) _____

(OVER)

11) Which of the following expressions is equivalent to

$$\frac{a^{-2}(b^2c^3)^{-3}}{(a^{-5}b^{-4})^2\,c^{-7}}\;?$$

      11)_____

A) $a^8b^2c^2$ B) $\dfrac{a^8c^2}{b^2}$ C) $\dfrac{a^8b^2}{c^2}$ D) $\dfrac{b^8c^2}{a^2}$

      E) None of the above

12) A car leaves Wilkes-Barre traveling at a constant speed of 57 mph. Forty five minutes, later a second car leaves from the same place and travels at a constant speed of 64 mph along the same road. How long will it take for the second car to catch up with the first?

      12) _____ hrs____

13) Suppose $f(x) = \arcsin x\; -\; 2x$, $g(x) = \cos x$, and $h(x) = x + \pi$. Find $(f \circ g \circ h)(\frac{\pi}{4})$.

      13) _____

14) The value of x in the triangle below is

      14)_____
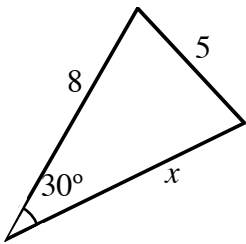
A) $\sqrt{73}$ D) the value is unique, but it is not A - C

B) 7   E) it cannot be uniquely determined
               given this information

C) 10

15) Find all real numbers x such that $e^{2x} - 3e^x + 2 = 0$

      15) __$x =$_____

16) How many real zeroes does the polynomial $p(x) = x^6 + x^4 + x^2 + 1$ possess?

      16) _____

17) What is the minimum value of the function $f(x) = \sqrt{3}\,x^2 + x + 4.$

      17) _____

18) Solve for p in terms of q if $\log p + \log q = \log(p + q)$ where $q \neq 1$.

      18) __$p =$_____

19) A quiz has 4 multiple-choice questions with each question having 4 choices. Suppose a student guesses on all questions. What is the probability that the student gets at least one question correct?

      19) _____

20) Find all ordered pairs that satisfy the system:

$$x^2 + xy = 1$$
$$y^2 + xy = 8$$

      20) _____

LUZERNE COUNTY MATHEMATICS CONTEST
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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What value of c makes $x = 8$ and $x = -4$ solutions to the equation $x^2 - 4x + c = 0$?

1) ___$c =$_____

2) A student has earned grades of 75, 82, 71, and 84. What score must the student receive on the next exam to attain an average of 80 ? (All tests are weighted equally)

2) _____

3) Find the length of x if the shaded area is 90 in^2 .

3) ___$x =$_____ in_____
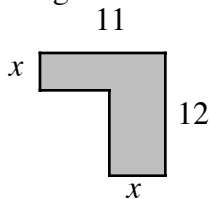
4) How many distinct solutions does the equation $2 \cos 4\theta = 1$ have on the interval $[\, 0, \frac{3\pi}{2} \,]$?

4) _____

    A) 0   B) 2   C) 4   D) 6   E) 8

    F) an infinite number of distinct solutions

5) Find the distance in the plane between the point $P = (\, 0, 8 \,)$ and the point $Q = (-4, 2)$.

5)_____

6) A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. Assuming the farmer wants to maximize the area, how long is the piece of fence parallel to the river?

6) _____ ft_____

7) In quadrilateral $ABCD$, $AB \perp BC$ and $DC \perp BC$. $\overline{AB} = 4$, $\overline{DC} = 3$, and $\overline{BC} = 2$. Find the area of the quadrilateral.

7) _____

8) How many distinct strings can be formed from permutations of the string $ALABAMA$?

8) _____

    A) 5040   B) 35   C) 210   D) 840   E) None of the above

9) Find all values of x such that $\dfrac{x + 1}{2x - 3} > 0$.

9) _____

10) Completely factor the polynomial $(x^2 + 3)^2 - 6(x^2 + 3) + 8$.

10) _____

(OVER)

11) Assume you roll two fair six-sided dice. What is the probability the sum of the dice is greater than 8?

11) _____

12) How many distinct real numbers x satisfy the equation below?

12) _____

$$x^2 - 8x + 5 - \cos^2 x \; = \; (x - 4)^2 - 11 + \sin^2 x$$

A) none   D) four
B) two   E) all real numbers
C) one

13) A six-foot tall man is walking away from a lamp post that has its light source 20 feet above the ground. How far from the lamp post is the man when the shadow is twelve feet long?

13) _____ ft _____

14) If $\sin T = \dfrac{-4}{9}$, find $\tan T \cot T + \csc T$.

14) _____

15) Which of the following cubic polynomials with integer coefficients have roots at $x = 3$, $2 + i$, and $2 - i$ respectively?

15) _____

A) $x^3 - 3x^2 + 5x - 15$ B) $(3x^2 + 15)(x - 3)$
C) $x^3 - 7x^2 + 17x - 15$ D) all of the above

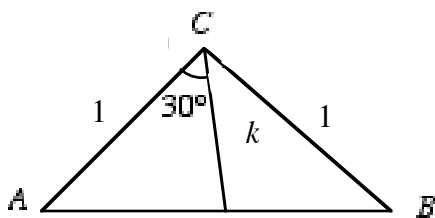**16)** Find all values x such that $5 - |4x + 1| \le 2$.

16) _____

17) Find the rectangular coordinates for the point whose polar coordinates are given by $(8, \dfrac{5\pi}{3})$.

17) _____

18) In triangle ABC below, the line segment k bisects angle C. Find the length of k.

18) _____

19) Evaluate $15^4 - 6^4$

19) _____

20) The locus of points defined by the equation
$5x^2 - 10x + 6y^2 + 36y = -34$ defines

20) _____

A) a non-circular ellipse   B) a circle

C) an hyperbola   D) a point   E) none of the above as a subset of the plane