LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2006 Junior Examination
(Section I)

NAME: ______________________________ **Address:** ______________________________

SCHOOL: ______________________________ **City/ZIP:** ______________________________

Telephone: ______________________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Simplify $\dfrac{a^2b^{-8}c^4}{a^{-3}b^5(c+1)}$ **1)**____________________.

2) Find the length of the unique line segment between the points (1, 0) and (3, 6). **2)**____________________.

3) $\sin^2 x =$ **3)**____________________.

(a) $1-\cos^2 x$ **(b)** $\sin^2(x+4\pi)$

(c) $\cos^2(\frac{\pi}{2}-x)$ **(d)** each of **a** - **c** is true

(e) none of **a** - **c** is true

4) Find all values of k such that $kx^2+kx+8=0$ has exactly one real solution. **4)** $k =$____________________

5) Given two functions f and g in which $f(1)=3, f(3)=1, g(1)=5$ and $g(3)=3$, what is the value of $g(f(g(3)))$? **5)**____________________

6) What is the volume of a sphere whose diameter is 10? **6)**____________________

7) Find all values of x such that $4-|3x+5|\le 2$. **7)**____________________

8) A student's grade on three examinations are 92, 71, and 53 respectively. If the student's last examination is weighted twice as much as any of the prior three exams, what is the minimum integer grade the student must get on this exam to obtain an average of at least 70? **8)**____________________

9) 2004 was a leap year. July 4, 2004 was a Sunday. In what year will July 4^{th} next fall on a Sunday? **9)**____________________

(a) 2008 **(b)** 2009 **(c)** 2010 **(d)** 2011 **(e)** 2012

10) Find the domain of the function $f(x)=\dfrac{x+1}{\sqrt{1-x^2}}$. **10)**____________________

(OVER)

11) Find all solutions to the equation $2\cos^2 x - 5\cos x + 2 = 0$ on $[0, 2\pi]$. **11)**________________________

12) Find a polynomial of degree 3 that has zeros 2, -3 and 4 in which the coefficient of x^2 is 2. **12)**________________________

13) If $f(\sin x) = \sin 3x$ for all real numbers x, compute $f(\cos(\frac{\pi}{6}))$. **13)**________________________

14) Which of the following relations possess x-axis symmetry? **14)**________________________

(a) $x^2 + y^2 = 1$ **(b)** $f(x) = 0$ **(c)** $y = x^2$
(d) both **a** and **b** **(e)** each of **a** - **c**

15) Find all real solutions to the inequality $2 \le \log_3 x \le 4$. **15)**________________________

16) How many subsets of size 3 does the set {1, 2, 3, 4, 5, 6} possess? **16)**________________________

17) Find $(f \circ g \circ f)(x)$ if $f(x) = 1 + x^2$ and $g(x) = e^x$. **17)**________________________.

18) Suppose a sequence of numbers is given by **18)**________________________

20, 23, 21, 24, 22, 25, 23, 26, 24, ...

What is the 50^{th} term in the sequence?

19) A rhombus $ABCD$ has diagonals of length 10 and 12 respectively. Find the area of the rhombus. **19)**________________________

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20) Find all values of m for which the equation $(m^2 + 2m + 3)x = 3(x + 2) + (m - 4)$ has a unique solution for x in terms of m. **20)**________________________

LUZERNE COUNTY MATHEMATICS CONTEST

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Wilkes University - 2006 Junior Examination
(Section II)

NAME: ______________________ **Address:** ______________________

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Telephone: ______________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Express $\sin(330°)$ as a rational number. **1)**______________________

2) Find the equation of the line parallel to the line $3x - 7y = 10$ that passes through the point (0, 1). **2)**______________________

3) Assume x is directly proportional to y. When $x = 10$ the value of $y = 5$. What is the value of y when $x = \frac{20}{3}$? **3)**______________________

(a) $\frac{3}{10}$ **(b)** $\frac{10}{3}$ **(c)** $\frac{40}{3}$ **(d)** $\frac{3}{40}$ **(e)** none of the above

4) Given the following system of linear equations **4)** $y =$ ______________________

$$2x - 4y = 10$$
$$4x + y = 29$$

Find the value of y.

5) What is the greatest common divisor of 360 and 405? **5)**______________________

6) Find all real solutions to the inequality $3 - 5x \leq 1 + 2x$. **6)**______________________

7) Suppose $f(x) = Ax^2 + 2x + 5$. Determine A so that $f(1) = f(-2)$. **7)** $A =$ ______________________

8) Assume the area of an equilateral triangle is $72\sqrt{3}$ square units. Find the length of an arbitrary side of the triangle. **8)**______________________

9) Evaluate $\frac{32^6}{16^6}$. **9)**______________________

10) Given that $\frac{1}{F} = \frac{1}{u} + \frac{1}{v}$, solve for v in terms of F and u. **10)** $v =$ ______________________

(OVER)

11) Find the remainder when $x^4 + 3x^3 + 2$ is divided by $x^2 + 1$. **11)**________________________

12) Convert the Cartesian coordinates of the point $P = \left(4, 4\sqrt{3}\right)$ to polar coordinates (r, θ), where θ is in $(-2\pi, 0]$. **12)**________________________

13) Assume the operation $\otimes$ is defined on real numbers as follows: **13)**________________________

$$x \otimes y = 2^y x$$

How many ordered pairs (x, y) satisfy the equation $x \otimes y = x$?

(a) 0 **(b)** 1 **(c)** 2 **(d)** there are an infinite number of such pairs
(e) none of the above.

14) Find B such that $\dfrac{x}{x^3 - 2x^2 + x} = \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$. **14)** $B =$ ____________________

15) Compute $\left(\sqrt{3} - i\right)^6$. **15)**________________________

16) For $x < -1$, $\sqrt{x^2 + 2x + 1} =$ **16)**________________________

(a) $x + \sqrt{2x} + 1$ **(b)** $x + 1$ **(c)** $-1 - x$
(d) each of **a** - **c** is true **(e)** none of **a** - **c** are true

17) Consider a triangle ABC where $\left|\overline{AB}\right| = 10$, $\left|\overline{BC}\right| = 16$, and $\angle ABC = \dfrac{2\pi}{3}$ radians. Find $\left|\overline{AC}\right|$. **17)**________________________

18) Compute $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \ldots + \dfrac{1}{100 \cdot 101}$ **18)**________________________

19) Find the smallest integer value of k so that if $x > k$, then $\left(\dfrac{1}{2}\right)^x < 0.0001$. **19)** $k =$ ____________________

20) How many distinct strings can be made by permuting the letters in the string *ALABAMA*? **20)**________________________