

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2007 Junior Examination

(Section I)

NAME: _____

Address: _____

SCHOOL: _____

City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the circumference of a circle whose radius is 5 units. 1) _____

2) Find the amplitude of the following trigonometric function: 2) _____

$$f(x) = \frac{5}{2} \sin(3x + \pi).$$

3) Find the fraction, in lowest terms, whose decimal representation is $0.\overline{92}$. 3) _____

4) If $2 + 4 + 6 + \dots + 2n > 72$, the smallest possible value of the integer n is _____. 4) $n =$ _____

5) Find all the real values of k such that $8x^2 + kx + k = 0$ has 2 distinct real solutions. 5) _____

6) Find all real numbers x which satisfy the following inequality: 6) _____
 $x^2 - 4x \leq 2x - 5$.

7) The constant in the expansion of $\left(\sqrt[3]{x} + \frac{1}{\sqrt[5]{x}}\right)^8$ is _____. 7) _____

8) Express the following as an integer: $3\left(\sqrt{\sqrt{\sqrt{2^{16}}}}\right)$ 8) _____

9) For $y \neq 2x$, $\frac{2x}{2x - y} + \frac{y}{y - 2x}$ equals 9) _____

(a) 1 (b) -1 (c) $2x + y$ (d) $x + y$

10) A merchant has 8 pounds of coffee worth \$2.50 per pound. He wishes to add premium coffee worth \$4 per pound and regular coffee worth \$2 per pound to this mixture in order to obtain 50 pounds of a mixture that is worth \$3.00 per pound. How much premium coffee should he add? 10) _____ pounds

(OVER)

11) Assume $x \neq 0$. Find A , B , and C such that .

$$\frac{2x}{x^3 + 2x^2 + x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

11) $A=$ _____ $B=$ _____ $C=$ _____

12) Find the domain of $\frac{3}{\left(\frac{\sqrt{x+5}}{7-x}\right)}$.

12) _____

13) Given $N = 2^6 \cdot 5^7$, how many divisors of N are multiples of 10?

13) _____

14) If $A = \{x \mid \sqrt{x-2} \leq 0\}$ and $B = \{x \mid 10^{x^2-2} = 10^x\}$, then $A \cap \bar{B}$ is (where \bar{B} denotes the complement of the set B)

14) _____

- (a) $\{2\}$ (b) $\{-1\}$ (c) $\{x \mid x \leq 2\}$ (d) \emptyset

15) Find a polynomial of degree two (2) with integer coefficients and leading coefficient two (2) in which one of the roots is $(1+i)$.

15) _____

16) Suppose the point $P(-3, y)$ and the point $Q(x, \sqrt{5})$ are symmetric about the x -axis. What are the values of x and y ?

16) $x =$ _____ $y =$ _____

17) Write $\cos 3x$ in terms of $\cos x$.

17) _____.

18) If $A = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ and $B = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$, then express $\sqrt{A^3 + B^3 - 367}$ as an integer.

18) _____

19) $\sec^2(\text{Arc tan } x) =$

19) _____

- (a) x^2 (b) $\sec x \tan x$
(c) $x^2 + 1$ (d) none of the above

20) If the straight line $y = x + m$ intersects the curve $\sqrt{1-y^2} = x$ exactly two times, then m will be contained in which of the following intervals?

20) _____

- (a) $(-\sqrt{2}, \sqrt{2})$ (b) $(-\sqrt{2}, -1]$
(c) $(-\sqrt{2}, 1]$ (d) $[1, \sqrt{2})$

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2007 Junior Examination
(Section II)

NAME: _____ Address: _____
SCHOOL: _____ City/ZIP: _____
Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) Find the slope of any line parallel to $8x - 3y = 24$. 1) _____
- 2) Find all values of x such that the distance between $(x, 4)$ and $(-2, 6)$ is 3. 2) _____
- 3) Find the vertex of the parabola $x = y^2 + 6y + 11$ 3) _____
- 4) Two cars are stopped at a rest area. The first car leaves the rest area at 3pm traveling south at 60 mph. The second car leaves the rest area 12 minutes later and travels 70 mph along the same path as the first car. At what time does the second car catch the first car? 4) _____
- 5) Bob wants to divide $4\frac{1}{8}$ pounds of candy equally among his 12 friends. How many pounds of candy does each of Bob's friends receive? 5) _____ pounds
- 6) Find the exact value of $\cos(x + y)$ if $\sec x = \sqrt{2}$, $\cos y = \frac{1}{2}$, and both x and y are quadrant I angles. 6) _____
- 7) Find all the real solutions to $\sqrt{4 - x^2} < x + 1$. 7) _____
- 8) Find all values of t such that the line segment passing between the points $(-3, 5)$ and $(t, t + 1)$ has a slope of $-t$. 8) _____
- 9) Find all real solutions to $\frac{1}{x^{\frac{8}{9}}} = x^{2x}$. 9) _____
- 10) How many ways are there to place 20 indistinguishable balls into 4 distinct urns if each must contain at least 2 balls? 10) _____

(OVER)

11) Assume we flip 5 fair coins. What is the probability that 1 or 2 of the coin's top faces are heads? 11) _____

12) Let $n = x - y^{x-y}$. What is n if $x = 2$ and $y = -2$? 12) _____

13) $\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{9} + \sqrt{10}} = ?$ 13) _____

14) Find the sum of the integers between 20 and 100 inclusive. 14) _____

15) Let $x \in [0, \pi]$. The number of solutions to the equation $\cos 7x = \cos 5x$ is _____. 15) _____

16) If $2 < x < 5$, then $\sqrt{(x - 2)^2} + |x - 5| =$ 16) _____

17) How many solutions does $\frac{\pi}{2} \sin x = x$ possess? 17) _____
(a) 1 (b) 2 (c) 3 (d) infinitely many (e) none

18) If $0 < b < a < 1$, then which of the following are true? 18) _____
(a) $ab < b^2 < 1$ (b) $\log_{\frac{1}{2}} b < \log_{\frac{1}{2}} a < 0$
(c) $2^b < 2^a < 2$ (d) $a^2 < ab < 1$

19) If $a - b < 0$, which of the following must be true? 19) _____
(a) $a > b$ (b) $ab > 0$ (c) $\frac{a}{b} < 0$ (d) $-a > -b$

20) Consider the quadrilateral below with the indicated vertices. The quadrilateral 20) _____
(a) is not a parallelogram. (b) is a parallelogram.
(c) is a rhombus, but not a square. (d) is a square.

