

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2008 Junior Examination

(Section I)

NAME: _____

Address: _____

SCHOOL: _____

City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) $16 + 8 \div 4 \cdot 2 = ?$.

1) _____

(a) 20

(b) 12

(c) 17

(d) none of the above

2) What is the circumference of a circle with a radius of 4 units?

2) _____

3) Find the surface area of a right circular cylinder with radius 2 and height 4 units .

3) _____

4) Find the vertex of the parabola $y^2 - 6y - x + 16 = 0$.

4) _____

5) Find all real values of k such that the line segment passing through the points (3, 4) and (-5, 2) is perpendicular to the line segment passing through the points (-6, 1) and (k , 7) .

5) _____

6) Find the 57th term of the sequence 12, -4, 17, -9, 22, -14, ...

6) _____

7) A square and a circle have equal perimeters. What is the ratio of the area of the circle to the area of the square?

7) _____

8) Suppose a , b and c are real numbers such that $ab = 7$ and $bc = 8$. If $a + c = 5$, then $b = ?$

8) $b =$ _____

9) If $f(x) = a^x$ where $a > 0$, express $\frac{f(x+1) - f(x)}{f(x)}$ as a function of a which does **not** depend on x .

9) _____

10) The three digit number $2a3$ is added to 326 to give the three digit number $5b9$. If $5b9$ is divisible by 9, what is $a + b$?

10) _____

(OVER)

- 11) If the sum of the first $3n$ positive integers is 150 more than the sum of the first n positive integers, then the sum of the first $4n$ positive integers is
 (a) 300 (b) 350 (c) 400 (d) 450 (e) 600 11) _____
- 12) If $\frac{|a|}{a-a^2} = \frac{1}{a-1}$, then a will satisfy
 (a) $a > 0$ and $a \neq 1$ (b) $a \leq 0$
 (c) $a \neq 0$ and $a \neq 1$ (d) $a < 0$ 12) _____
- 13) Suppose $\sin(\theta) = \frac{x-1}{x+1}$ where $0 \leq x \leq 1$. Find all real solutions x to $\tan\theta = -3\sqrt{x}$. 13) _____
- 14) If x and y satisfy $(x+5)^2 + (y-12)^2 = 14^2$, then the minimum of $x^2 + y^2$ is
 (a) 2 (b) 1 (c) $\sqrt{3}$ (d) $\sqrt{2}$ 14) _____
- 15) Assume we throw two fair dice. What is the probability that the sum of the top faces is less than or equal to 5? 15) _____
- 16) If in a sequence $\{x_n\}_{n \geq 1}$, $x_1 = 1$ and $x_{n+1} = \frac{\sqrt{3}x_n + 1}{\sqrt{3} - x_n}$, then $x_{1999} - x_{601} = ?$ 16) _____
- 17) If $4x^2 - 6x + m = (x-3)(ax+b)$ where a, b , and m are real numbers, what is the value of m ? 17) $m =$ _____
- 18) Find all real solutions x to $\sin 3x = \frac{1}{2}$, where $0 \leq x \leq \frac{\pi}{2}$. 18) _____
- 19) Determine all real solutions to the equation $\sqrt{x+4} + \sqrt{x+6} = 2$. 19) _____
- 20) If x_1 and x_2 are real solutions to $x^2 + kx + 5(k-5) = 0$ and $2x_1 + x_2 = 7$, find all real values for k . 20) _____

LUZERNE COUNTY MATHEMATICS CONTEST

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Wilkes University - 2008 Junior Examination
(Section II)

NAME: _____ **Address:** _____
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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) A right triangle has legs of length 12 and 16 units respectively. What is the perimeter of the triangle? **1)** _____
- 2) Find all real numbers x which satisfy $x^2 + 5x < -6$. **2)** _____
- 3) Find the equation of the line that is a perpendicular bisector of the line segment with endpoints (5, 2) and (-3, 6). **3)** _____
- 4) Write $\frac{(.02)^5}{(.03)^3}$ as a fraction in lowest terms. **4)** _____
- 5) Find the area of a triangle with vertices (-3, -5), (2, 3) and (4, -5) **5)** _____
- 6) Convert $(2, \frac{5\pi}{3})$ from polar to rectangular coordinates. **6)** _____
- 7) If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$, then $(f \circ g \circ f)(x) =$ **7)** _____
 - (a) $\sqrt{x^2 + 1}$ for all x
 - (b) $\sqrt{x^2 + 1}$ for $x \geq 0$
 - (c) $\sqrt{x + 1}$ for $x \geq 0$.
 - (d) $\sqrt{x + 1}$ for all x
- 8) If $f(x) = |x - 1| - |x|$, then $f\left(f\left(\frac{1}{2}\right)\right) = ?$ **8)** _____
- 9) Express $\frac{i^4 + i^2}{i + 1}$ as a real number. **9)** _____
- 10) Let f be an even function and g be an odd function such that f and g are defined for all x . Which of the following are even functions? **10)** _____
 - (a) $f \circ g$
 - (b) $g \circ f$
 - (c) $g \circ g$
 - (d) both (a) and (b)
 - (e) both (b) and (c)

(OVER)

11) Suppose $x \geq 1$. Write $\frac{\sec^2(\arctan \sqrt{x^2 - 1})(\log_{10} 10,000) - 64^{\frac{2}{3}}}{e^{3 \ln(x+2)}}$ as a rational expression in x . 11) _____

12) Find the coefficient of $x^2 y^2 z^5$ in the expansion of $(-3x + y + z)^9$. 12) _____

13) A person buys 3 oranges for 10 cents. He sells 5 oranges for 20 cents. How many oranges must he sell to make a \$1 profit? 13) _____

14) What is the remainder when $x^{79} + 17x^{28} - 6x^{12} + x - 12$ is divided by $x + 1$? 14) _____

15) Express the solution x to the equation $\log_{10} x - 5 \log_{10}(3) = 2$ as an integer. 15) _____

16) Express $2\sqrt{3 - 2\sqrt{2}} + \sqrt{17 - 12\sqrt{2}}$ as an integer. 16) _____

17) How many of the first one hundred positive integers are divisible by all of the following numbers: 2, 3, 4, and 5? 17) _____

18) If $\frac{a-b}{a} = \frac{3}{5}$, then $\frac{a}{b}$ is 18) _____

- (a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $-\frac{2}{5}$ (d) $-\frac{5}{2}$

19) If $f(x) = \begin{cases} x^2, & x \leq 0 \\ -x^2, & x > 0 \end{cases}$, find the inverse function $f^{-1}(x)$. 19) $f^{-1}(x) =$ _____

20) Let r and s be two solutions to the equation $x^2 + x - 3 = 0$. The value of $r^3 - 4s^2 + 19$ is 20) _____

- (a) -4 (b) 8 (c) 6 (d) 0