Luzerne County Council of Teachers of Mathematics
Wilkes University – 1993 Senior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

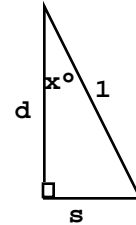1) If $(x - y)^2 = 40$ and $x^2 + y^2 = 60$, what is the value of xy?

2) Simplify $\sqrt{45} - \sqrt{20} + \sqrt{5}$

3) There is enough food at a picnic to feed 20 adults or 32 children. If there are 15 adults at the picnic, how many children can still be fed?

4) If $\sin \theta = \frac{3}{7}$ and $90° < \theta < 180°$, what is the value of $\cot \theta$?

5) A line contains the points (-2,1) and (4,4). At what point does it intersect the x-axis?

6) In the figure shown, if $\arcsin(s) = 2 \arcsin(d)$,
   then what is the value of x?

7) Express the following in terms of $\sin \theta$ and $\cos \theta$, and simplify your answer:
$$\frac{1}{\tan \theta + \cot \theta}$$

8) Evaluate: $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

9) Two ships leave from the same port at 11:30 a.m. If one sails due east at 20 miles per hour and the other due south at 15 miles per hour, how many miles apart are the ships at 2:30 p.m.?

10) If all S are M, and
   no P are M,
   then we can conclude which of the following:
   (a) All S are P
   (b) All M are S
   (c) Some S are not M
   (d) Some M are P
   (e) No P are S

11) If the angles of a triangle are in the ratio 2:3:4, What is the measure of the largest angle? Express your answer in degrees.

12) Determine the value(s) of K for which the line $x = K$ is tangent to the circle $x^2 + y^2 - 4x + 2y + 1 = 0$.

13) If two dice are rolled, what is the probability that the number of dots on the two exposed faces will total 4?

14) A gallon of water is added to 6 quarts of a solution which is 50% acid. What percent of the new solution is acid?

15) In the figure shown, both the circle centered at P and the circle centered at Q have a radius of 1. What is the area of the shaded region?
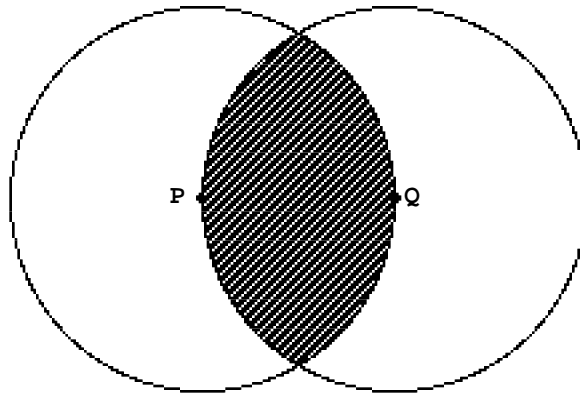
16) If $x^2\,ax + bx + ab = 0$ and $x + b = 2$, then what is the value of $x + a$?

17) Find all values of x which satisfy $|5x - 16| = |2x + 2|$.

18) Suppose f is a function satisfying $f(x)f(y) - f(xy) = x + y$ for all real numbers x and y. Determine the formula for $f(t)$.

19) The points (-2,4), (3,4) and (3,-2) are connected to form a triangle. What is the area of the triangle?

20) In a group of 40 students, 25 applied to Wilkes University and 30 applied to King's College. If 3 students applied to neither Wilkes nor King's, how many students applied to both schools?
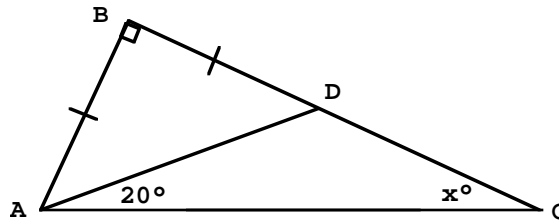
(Section II)

1) What is the value of $\log_3 \sqrt{3}$?

2) Find all values of x for which $9^{\log_3(x^2)} = 16$.

3) If $f(x) = x^2 + bx + 1$ and $f(1) = 4$, what is the value of b?

4) In the figure shown, ABD is a right isoceles triangle. Find x.

5) For which values of a will the quadratic equation $4x^2 + ax + 9 = 0$ have only 1 solution?

6) The sum of Alan's age and Barbara's age is 40.
The sum of Barbara's age and Carl's age is 34.
The sum of Alan's age and Carl's age is 42.
How old is Barbara?

7) Find all points on the x–axis which are exactly 3 units away from the point (5,-1).

8) Determine all values of x in the interval $[0,2\pi)$ such that $2\sin^2 x + \sin x = 1$.

9) A gasoline tank is $1/4$ full. After adding 10 gallons of gasoline, the gauge indicates that the tank is $2/3$ full. Find the capacity of the tank in gallons.

10) The vertical position p of a projectile at time t is given by $p = -16t^2 + 64t + 80$. What is the maximum height that the projectile will attain?

11) Solve for x: $\dfrac{5}{x+4} = \dfrac{7}{x+6}$

12) A student taking a true–false test guesses randomly on three items. What is the probability that exactly two of the guesses will be correct?

13) Determine all values of x which satisfy $x^2 < 4x + 5$.

14) Find all values of x for which $\log_2(\log_2(x^2)) = 4$.

15) If $f(x) = 3x - 2$ and $g(f(x)) = x$, then find $g(x)$.

16) In the figure shown, a small square is inscribed inside a circle, which is itself inscribed inside a large square. Find the ratio of the area of the large square to the area of the small square.
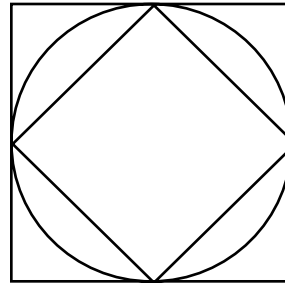
17) The average of 7 consecutive even integers is 14. What is the sum of the first two integers?

18) If for all n, $2^n + 2^n + 2^n + 2^n = x(2^{n+1})$, then what is the value of x?

19) Find all values of x for which $2x^{2/3} - x^{1/3} - 1 = 0$.

20) The average cost of 13 items in a display case is $12. After one of the items is removed, the average cost of the 12 remaining items is $11. What was the cost of the removed item?