LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University – 1994 Senior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Solve $\dfrac{n-2}{n+3} = \dfrac{3}{8}$

2) Find the greatest common divisor of the numbers 28, 35, and 56.

3) Solve the equation $2^{x^2-1} = 64$.

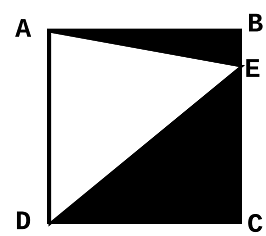
4) Find all x in the interval $[0,2\pi)$ such that $|\cos x| = \dfrac{\sqrt{2}}{2}$.

5) If $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$

then what is the value $f \circ f(\pi)$?

6) Evaluate $\lim\limits_{x \to 2} \dfrac{x^4 - 2x^3 + x - 2}{x^3 - 8}$

7) If $ABCD$ is a square of side length 6 inches, then what is the area of the shaded region?



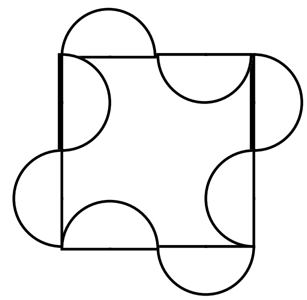
8) Express $\dfrac{\dfrac{y}{x^2} - \dfrac{x}{y^2}}{\dfrac{y}{x} - \dfrac{x}{y}}$ as a simple fraction.

9) A rhombus has a side length of 4 inches and an angle measure of 60 °. What is the area of the rhombus?

10) A woman has only nickels and dimes in her change. She has eleven more nickels than dimes. If she has a total of \$2.65 in change, how many dimes does she have?

11) Flying against a headwind, an airplane can fly 3,000 km in 6 hours. At the same air speed, it can make the return flight in a tailwind in 5 hours. What is the windspeed.

12) In the figure shown, the curved path is made up of eight semicircles of equal diameter. If the total length of the curved path is 16π inches, then find the area of the square.



13) Solve the equation $\log_3 x + \log_3(x - 8) = 2$.

14) How many inflection points does the graph of $f(x) = ax^2 + bx + c$ have, where a, b, and c are constants and $a \neq 0$?

15) If $\sec\theta = -\frac{5}{3}$ and $\sin\theta < 0$, find $\tan\theta$.

16) Given that $f(x) = 0$ only for $x = -1$ and $x = 2$, and that $g(x) = 2x - 1$, find all x such that $((f \circ g)(x) = 0$.

17) Find a complex number in the form $a + bi$ such that, when multiplied by $(1 + 2i)$, the result is 1.

18) What is the maximum value of the slope of a tangent line to the graph of $y = \sin x$, where x is in radians?

19) The statement $x^2 + x + 1 \geq \frac{3}{4}$

(a) is true for all real numbers x.
(b) is false for all real numbers x.
(c) is true for some but not all real numbers x.
(d) cannot be determined from the information given.

20) Sam has one of each of the following: a penny, nickel, a dime, a quarter, a half dollar, and a dollar bill. He will definitely place a bet on his next poker hand. how many amounts are possible for Sam to place as his next bet?

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(Section II)

1) Doug's scores on his first three math tests are 86, 88, and 78. If he wants to average at least 80 after his fourth test, what is the minimum score he must obtain on the fourth test?

2) Solve the equation $3 = 2\sqrt{x} + x$.

3) One side of a triangle is three inches long and the second side of the triangle is five inches long. If x denotes the length of the the third side in inches, what are all possible values of x?

4) Find the least common multiple of the numbers 30, 40, and 70.

5) Give the radian measure of a 330° angle.

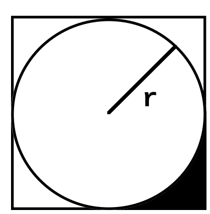
6) Find all values of x for which the following inequality holds:

$$\frac{x - 2}{x + 1} < 0$$

7) The sum of three consecutive odd integers is 81. Find the three numbers.

8) Fahrenheit and Celsius temperature are related by the formula $C = \frac{5}{9}(F - 32)$. If the temperature in degrees Celsius ranges over the interval $20 \le C \le 50$ on a certain day, what is the temperature in degrees Fahrenheit that day?

9) Express the area of the shaded region in terms of r, given that the circle is inscribed in the square.



10) What is the domain of the function

$$f(x) = \frac{x + 2}{x^3 + x^2 - 4x - 4}\ ?$$

11) Solve the equation: $4^{5x+2} = 16^3$

12) The center or a circle lies in the second quadrant and is 1 unit from the y–axis and 2 units from the x–axis. If the circle is tangent to the y–axis, find the equation of the circle.

13) Determine the period of the function $f(x) = \sin(6x - \pi)$.

14) There are eight people in a room and each person wishes to shake hands with every other person exactly one time. How many hand shakes are required?

15) If $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ -x^2 + 2kx - k^2 + 1 & \text{if } x > 0 \end{cases}$

then for what values of k is f continuous at $x = 0$?

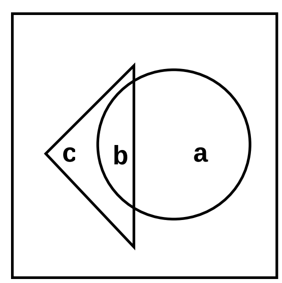
16) Assuming m and n are positive real numbers, rewrite the following expression as a single logarithm with coefficient 1: $\frac{1}{2}\log_5 m + \frac{1}{3}\log_5 n^4 - \log_5 m^2 n$

17) In the figure, let

S denote the set of all points inside the square,

T the set of all points inside the triangle,

and C the set of all points inside the circle.



Which of the following are true?

(a) $T \subset C$ (c) $a \notin T$ (e) $b \in T \cap C$

(b) $T \subset S$ (d) $a \notin S$ (f) $a \in C \cup T$

18) Each side of an equilateral triangle is four inches longer than the side of a square. The sum of the perimeters of the two figures is 96 inches. How long is each side of the triangle?

19) A multiple choice test consists of 5 questions, each with 3 possible answers. If a student guesses randomly on each question, what is the probability that she answers all questions correctly?

20) Find $f(x)$ if $f(x + 1) = x^2 + 3x + 5$.