Luzerne County Council of Teachers of Mathematics
Wilkes University – 1995 Senior Examination

(Section I)

**Directions:** Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Find the coordinates of the point of intersection of the lines $y = \frac{1}{2}x - 1$ and $y = -4x + 3$.

2) Solve for $x$: $\dfrac{x}{x-1} = \dfrac{8}{x+2}$

3) Determine the amplitude of the function $y = 3 \sin 2x$.

4) Your combined federal and state taxes amounts to 35% of your gross pay. What is the minimum gross pay, rounded to the nearest thousand dollars, that you must earn in order to have a net pay of at least $40,000?

5) The statement $2x^2 - 2x + 1 < \frac{1}{2}$

    a) is true for all real numbers $x$

    b) is false for all real numbers $x$

    c) is true for some, but not all real numbers $x$

    d) cannot be determined from the information given

6) One candle will burn completely in 4 hours, while a second candle of equal length requires 5 hours to burn completely. If the candles are lit at the same time, how long will they burn before one of them is exactly three times as long as the other?

7) If $\log_a 2 = 0.3010$, $\log_a 3 = 0.4771$, and $\log_a 5 = 0.6990$ then find the value of $\log_a 0.27$.

8) Suppose $F'(x) = \sin \frac{x}{2}$ and $F(2\pi) = 3$. Determine the formula for $F(x)$.

9) In the square with side length $a + b$, the area of the shaded square is one third the total area. Determine the ratio $\dfrac{a}{b}$.
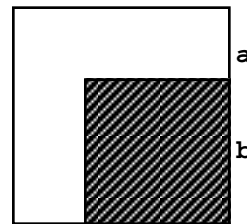


10) If $f(x) = 3x - 2$ and $g(x) = x - x^2$, then find the value of $f(g(2))$.

11) If an angle measures $\frac{\pi}{11}$ radians, what is the degree measure of the angle?

12) How many 5 character passwords are possible if the first 2 characters are capital letters (A through Z) and the last three characters are digits (0 through 9)?

13) Solve for $x$: $2^x = 4^{x+1}$

14) A circle is tangent to the line $y = 2x + 2$ at point (2,6) and has its center on the $x$–axis. Find the radius of the circle.

15) Find the domain and range of the function $f(x) = \dfrac{1}{1 + \sqrt{x^2 + 4x + 1}}$.

16) A rectangular dog pen is constructed from 120 ft. of fencing. If $x$ represents the width of the pen, express the area $A(x)$ as a function of $x$.

17) Find the equation of the parabola with the vertex (3,-5), opening downward, which passes through the point (4,-7). Express your answer in the form $y = a(x - h)^2 + k$.

18) Reduce the following expression to lowest terms:
$$\frac{u - \dfrac{1}{u}}{1 - \dfrac{1}{u^2}}$$

19) Suppose $f(x) = \sqrt{2x - 7}$. Find the equation of the line tangent to the graph of $f$ at the point corresponding to $x = 8$. Express your answer in point–slope form.

20) Express the area A of the shaded region in terms of r given that the circle has radius r and the square is inscribed in the circle.
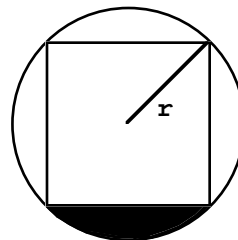
(Section II)

1) Solve the inequality: $2(x + 4) > 5x - 4$.

2) Find the equation of the line passing through (2,-3) and perpendicular $2x - 4y = 5$.

3) John is three years older than Nancy, and Nancy is twice as old as Norman. The average of their ages is 16. How old is Norman?

4) Find all values of $x$ for which the following inequality holds:
$$\frac{1 - x^2}{x + 3} < 0$$

5) Find all real solutions for the equation $x^4 + x^3 + 2x - 4 = 0$.

6) A deck 3 meters wide surrounds a rectangular pool which is twice as long as it is wide. Find the dimensions of the pool if the area of the deck is 360 m$^2$.
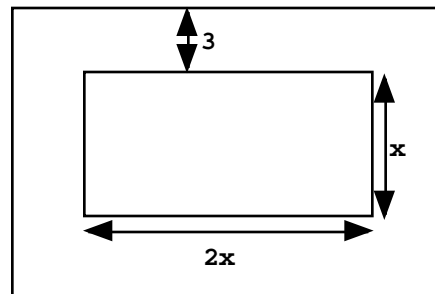


7) Solve the system of equations for the unknown quantities $x$ and $y$:

$$\begin{cases} 2x + 3y = 5 \\ -3x + y = -1 \end{cases}$$

8) If $f(x) = x^2 + 2x - 1$, and $h \neq 0$, find $\dfrac{f(x + h) - f(x)}{h}$ and simplify.

9) How many degrees are in an angle that is $\frac{1}{4}$ as large as its supplement?

10) Solve $\log_3 x + \log_3(x - 8) = 2$.

11) Find the greatest common divisor of the numbers 2100, 630, and 1470.

12) Solve the equation $3 = 2\sqrt{x} + x$.

13) Find $f(x)$ if $f(x + 2) = x^2 - 2x + 2$.

14) Evaluate $\lim\limits_{x \to 3} \dfrac{x^4 - 3x^3 - x + 3}{x^3 - 3x^2 + 3x - 9}$.

15) Consider a person standing near a streetlight. If the person is 6 ft. tall, standing 20 ft. from the base of the streetlight, and the person's shadow is 8 ft. long, how high is the streetlight?
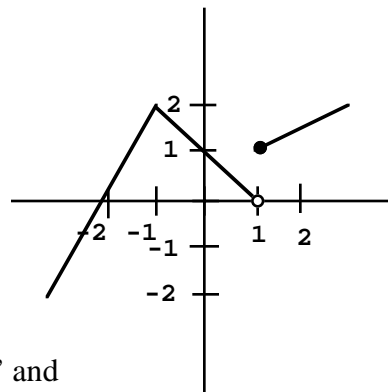
16) Find $\sin\left(2 \text{ Arcsin } \dfrac{3}{5}\right)$.

17) The negation of the statement "No dogs are purple" is:
   (a) All dogs are purple.
   (b) All dogs are not purple.
   (c) Some dog is purple.
   (d) Some dog is not purple.

18) Determine $x$ if $x^{\sqrt{\log_{10} x}} = 10^8$.

19) From the graph of $f$ given below, determine $\lim\limits_{x \to 1^-} f(x)$.



20) In how many ways can three books "a," "b," and "c" be placed on a shelf so that reading left to right, the three books will *not* be in alphabetical order?