Luzerne County Council of Teachers of Mathematics
Wilkes University – 1996 Senior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) The sum of three consecutive *even* integers is 246. Find the smallest of the three integers.

2) Simplify the expression $\dfrac{\dfrac{1}{x} - \dfrac{1}{x+1}}{1 + \dfrac{1}{x}}$.

3) Given $\ln a = 2$, evaluate the expression $\ln\left(\sqrt[4]{e^{\,3}a}\right)$.

4) Which of the following functions are even functions?
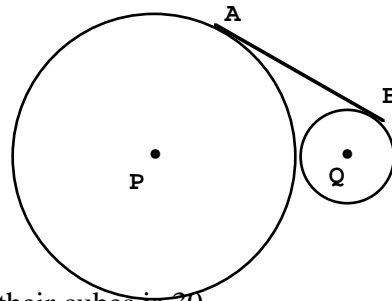
　　　a) $f(x) = \cos(5x)$

　　　b) $g(x) = \cos x - \sin x$

　　　c) $h(x) = \sqrt{x^{2} + 1}$

5) If a man 6 feet tall stands 10 feet from a 20 foot high lamp post, how long is the shadow of the man?

6) How many quarts of 75% alcohol must be mixed with 15 quarts of 45% alcohol to produce 70% alcohol?

7) The circles centered at P and Q have radii 6 and 2, respectively, and are tangent to each other. Find the length of their common external tangent $\overline{AB}$.

8) Find two numbers whose product is 5 and the sum of their cubes is 30.

9) If $\csc\theta = \dfrac{3}{2}$ and $\cos\theta < 0$, then find $\tan\theta$.

10) Water flows into a tank in such a way that the volume of water doubles each minute. If it takes 10 minutes to fill the tank, then how long does it take for the tank to become half full?

11) Find all complex numbers z, other than $z = 1$, satisfying the equation $z^3 = 1$.

12) Six white balls and four black balls are placed in a bag. If three balls are drawn at random without replacement, what is the probability that all three balls are white?

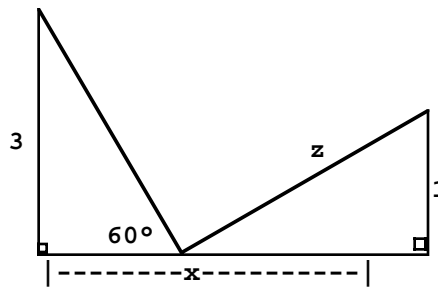13) If $F'(x) = \sin x \cos x$ and $F(0) = 1$, find $F(x)$.

14) Express the area A of the shaded region in terms of s, given that the square has side length s and the circle is inscribed in the square.
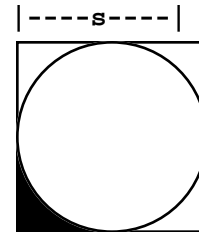
15) Express the equation $y = 2x^2 + 16x + 9$ in the form $y = a(x - h)^2 + k$.

16) How many right triangles are there with hypotenuse two units longer than one leg and one unit longer than the other leg?

17) Express z in terms x.

18) Find the domain of the function $f(x) = 3(\log_{10}(5x - 2))$.

19) An equation of a line tangent to the circle $x^2 + y^2 = 1$ that contains the point $(2, 1)$ is:

   a) $4x + 3y = 11$ c) $3x - 4y = 2$

   b) $4x - 3y = 5$ d) $3x + 4y = 10$

20) Among people who have a certain disease, the mortality rate is 30% for those who are not treated and the mortality rate is 5% for those treated. Suppose that among the people who have the disease, 80% have been treated. If a person is chosen randomly from those who have the disease, what is the probability that he will survive the disease?

(Section II)

1) Convert $\dfrac{5\pi}{12}$ radians into a degree measurement.

2) Find the equation of the line passing through (2, -1) that is perpendicular to the line passing through $(-\frac{1}{2}, 1)$ and $(2, \frac{3}{4})$.

3) Suppose A can do a certain job in 8 hours and B can do the same job in 5 hours. Working together, how long will it take them to complete this job?

4) Solve the following system of equations. Give your answer(s) in the form (x, y).
$$\begin{cases} x^2 + y^2 = 1 \\ x + 2y = 1 \end{cases}$$

5) The numerator of a fraction is 5 less than the denominator. If the numerator is doubled and the denominator is decreased by 4, the resulting fraction is $\frac{3}{2}$. Find the original the fraction.

6) Evaluate $\displaystyle\lim_{x \to 2} \dfrac{x^3 - 2x^2 + x - 2}{x^2 - 3x + 2}$.

7) Solve for x: $\ln(7 - x) + \ln(3x + 5) = \ln(24x)$

8) The given triangle has side lengths of 12, 12, and 8. Find the radius of the circle inscribed in the triangle.
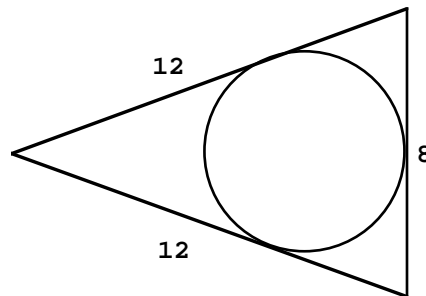
9) Find two positive numbers x and y having a ratio of 5 : 2 whose difference is 69.

10) Find a function $f(x)$ such that $f'(x) = f(x)$ and $f(1) = e^2$.

11) Find *all* values of x satisfying $-2 < \log_{10} x < -1$.

12) Find the domain of the function $f(x) = \dfrac{x^2 - x - 2}{x^3 + x^2 - x - 1}$.

13) Assume $\log_5 2 = 0.4307$ and $\log_5 3 = 0.6826$. Compute $\log_5 \left(\dfrac{6}{5} \right)$.

14) Convert the infinite, repeating decimal $0.\overline{24} = 0.242424\ldots$ into a fraction expressed in lowest terms.

15) If $f(x) = x^3$, evaluate and simplify the expression given by $\dfrac{f(1 + h) - f(1)}{h}$.

16) The statement $3x^2 - 6x + 4 \geq 1$

       a) is true for all real numbers x.

       b) is false for all real numbers x.

       c) is true for some but not all real numbers x.

       d) cannot be determined from the information given.

17) If $f(3x - 2) = x^2 - 1$, find $f(2)$.

18) Find A and B such that $\dfrac{3x + 1}{(x + 1)(x - 2)} = \dfrac{A}{x + 1} + \dfrac{B}{x - 2}$

19) $\angle A$ is the complement of $\angle B$. Twice $\angle B$ is the supplement of $\angle C$. If $\angle C$ measures $50°$, then what is the measure of $\angle A$?

20) Side $\overline{AC}$ of right triangle ABC is subdivided into 8 equal parts. From the point of subdivision, seven line segments parallel to $\overline{BC}$ are drawn to $\overline{AB}$. If $BC = 10$, find the sum of the lengths of the seven line segments.