

LUZERNE COUNTY MATHEMATICS CONTEST  
Luzerne County Council of Teachers of Mathematics  
Wilkes University – 1997 Senior Examination

(Section I)

**Directions:** Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Simplify:  $5\sqrt{20} - \sqrt{45} + 2\sqrt{80}$

2) Find two integers whose sum is 12 and whose difference is 18.

3) Solve for  $x$ :  $\frac{x-3}{2x+1} < 0$

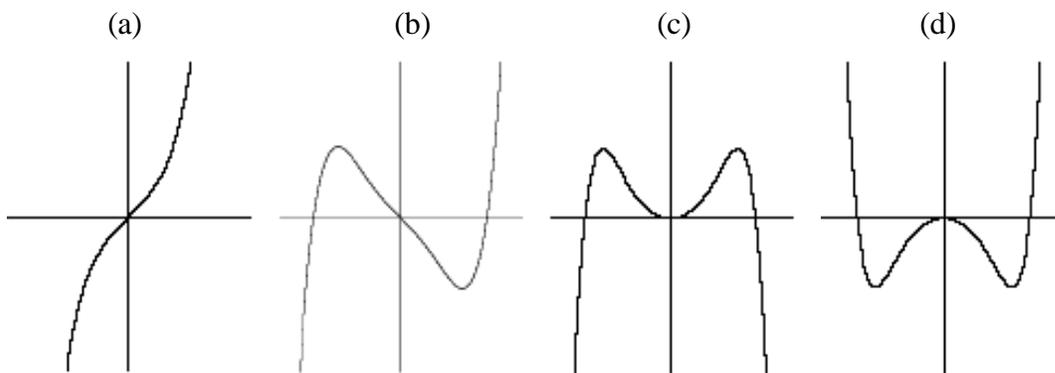
4) Determine the vertex of the parabola having the equation  $y = x^2 + 2x + 4$ .

5) Evaluate  $\lim_{x \rightarrow \infty} \frac{3x^2 + 3x - 2}{2x^2 + x + 100}$ .

6) Determine  $\sin \theta$  if  $0 < \theta < \frac{\pi}{2}$  and  $\tan \theta = 2\cos \theta$ .

7) Find  $A$  and  $B$  such that  $\frac{3x+1}{x^2-x-2} = \frac{A}{x+1} + \frac{B}{x-2}$ .

8) What is the best choice for the graph of  $f(x) = x^6 - x^4 - x^2$ ?

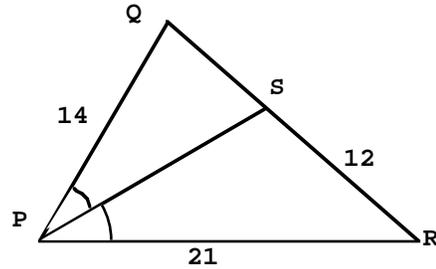


9) If  $a > 0$  and  $b > 0$ , solve for  $x$  and simplify:  $\log x = \frac{1}{2}(\log a + \log b)$

10) A plane flying parallel to the ground at an altitude of 500 ft passes directly overhead. Two seconds later, its angle of elevation is  $60^\circ$ . Assuming the plane's speed is constant, compute the plane's speed measured in feet per second.

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- 11) In the given triangle,  $m(\angle QPS) = m(\angle RPS)$ . If  $RS = 12$ ,  $PQ = 14$ , and  $PR = 21$ , determine  $QS$ .



- 12) Determine the domain of  $f \circ g$ , when  $f(x) = \sqrt{x}$  and  $g(x) = x^2 - x$ .

- 13) Find  $x$  and  $y$  so that both equations are satisfied:

$$\frac{4^x}{2^{x+y}} = 8 \text{ and } \frac{9^{x+y}}{3^{5y}} = 243$$

- 14) Find all real numbers  $x$  in the interval  $[0, 2\pi]$  satisfying  $\sin x = \cos x$ .

- 15) If  $f(x) = \frac{1}{1 + 10^{-x}}$ , find the inverse function  $f^{-1}(x)$ .

- 16) Determine the complex number  $a + bi$  such that  $(1 + i)(a + bi) = 1$ .

- 17) A ball is dropped from a height of 10 feet. On each bounce it rises to half its previous height. How far does it travel from the moment it is dropped until the moment it hits the ground for the sixth time?

- 18) Suppose that  $\tan x = \frac{2ab}{a^2 - b^2}$ , where  $0 < b < a$  and  $0 < x < \frac{\pi}{2}$ . What is  $\sin x$ ?

- 19) The sides of a right triangle are  $a$ ,  $a + d$ , and  $a + 2d$ , with  $a$  and  $d$  both positive. Determine the ratio  $a:d$ .

- 20) Team A has 11 wins and 9 losses. Team B has 20 wins and 8 losses. Assuming that A and B are the only teams involved, how many consecutive games must A win against B so the teams will have the same winning percentage?

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(Section II)

1) Convert the following angle given in radians to degrees:  $\frac{17\pi}{12}$

2) Solve for  $x$  and  $y$ : 
$$\begin{cases} 5x + 2y = 5 \\ 4x + 3y = 6 \end{cases}$$

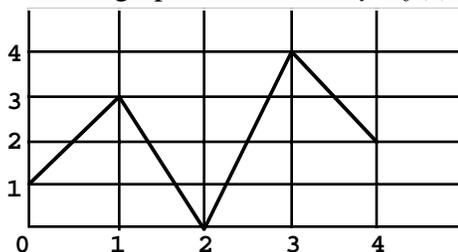
3) Suppose a line  $L$  passes through points  $(a,b)$  and  $(c,d)$ , where  $a \neq c$  and  $b \neq d$ . What is the slope of a perpendicular to  $L$ ?

4) Solve for  $x$ :  $x^2 + 3 \geq 4x$

5) What is  $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$  if  $f(x) = x^2$ ?

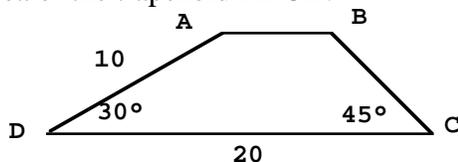
6) Represent the infinitely repeating decimal  $0.\overline{23} = 0.232323\dots$  as a ratio of two integers.

7) The sketch below shows the graph of a function  $y = f(x)$ . What is  $(f \circ f \circ f)(2)$ ?



8) Solve the for  $x$ :  $x - 10\sqrt{x} + 23 = 0$

9) Suppose  $ABCD$  is a trapezoid with  $AD = 10$ ,  $CD = 20$ ,  $m(\angle ADC) = 30^\circ$ ,  $m(\angle BCD) = 45^\circ$ . Determine the area of the trapezoid  $ABCD$ .



10) A company's profit (in thousands of dollars) by the function  $P(x) = 5 + 12x - x^2$ , where  $x$  is the number of units sold. What is the maximum possible profit?

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11) Let  $m_a$  denote the slope of the tangent line to the graph of  $f(x) = x^2$  at the point  $(a, a^2)$ . Determine all values of  $a$  for which  $m_a > 1$ .

12) If  $P = \frac{s}{(1+k)^n}$  then  $n$  equals:

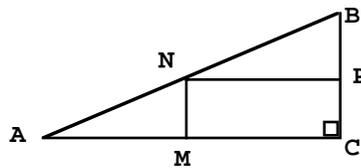
- a)  $\frac{\log\left(\frac{s}{P}\right)}{\log(1+k)}$       b)  $\log \frac{s}{P(1+k)}$       c)  $\log \frac{s-P}{1+k}$
- d)  $\log \frac{s}{P} + \log(1+k)$       e)  $\frac{\log s}{\log P(1+k)}$

13) Company A will rent you a car for \$10 per day. Company B will rent you a car for \$6 per day plus an initial fee of \$60. What is the minimum number of days for which it is cheaper to rent from company B?

14) Simplify the following expression:  $\left(\frac{(-x^2y)^3y^{-4}}{(xy)^5}\right)^{-2}$

15) Find all values for  $c$  for which it is possible to find two distinct real numbers whose sum is  $c$  and whose product is 100.

16) Suppose  $\triangle ABC$  has a right angle at  $C$ . Also suppose  $\overline{MN} \perp \overline{AC}$ ,  $\overline{NP} \perp \overline{BC}$ ,  $BC = 5$ , and  $AC = 12$ . If  $AM = x$  and  $MN + NP = y$ , express  $y$  in terms of  $x$ .



17) Find the center and radius of the circle with equation:

$$3x^2 + 3y^2 + 12x + 12 = 18y$$

18) The chemistry club at a certain school has 15 members. In how many different ways can a committee of 4 members be selected to plan a field trip?

19) Solve for  $x$ :  $\frac{11x+7}{x+2} < 6$ .

20) Find the formula for  $S_n$ , where  $S_n$  denotes the  $n$ th term of the arithmetic sequence 1, 6, 11, 16, 21, ...