

LUZERNE COUNTY MATHEMATICS CONTEST
Luzerne County Council of Teachers of Mathematics
Wilkes University – 1999 Senior Examination

(Section I)

Directions: Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for the problem.

1) Find all real values of x satisfying $\sqrt{x} + 6 = x$.

2) If $6\sin^2\theta - \sin\theta = 1$ and $0 \leq \theta \leq \pi$, then find θ .

3) Find all real numbers x such that $|18 - 2x| \leq 4$.

4) Evaluate $\lim_{x \rightarrow 3} \frac{x^3 + x^2 - 12x}{x^2 - 9}$.

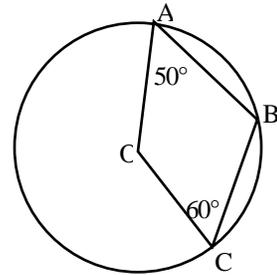
5) If $a \Delta b \Delta c = \frac{ab}{c}$, then which of the following equals 5?

a) $4 \Delta 3 \Delta 2$ b) $5 \Delta 2 \Delta 5$ c) $6 \Delta 4 \Delta 2$

d) $8 \Delta 4 \Delta 2$ e) $10 \Delta 2 \Delta 4$

6) If the area of a rectangle is 500 m^2 and its length is four times its width, then what are the dimensions of the rectangle?

7) In the figure shown, points A , B , and C lie on the circumference of the circle centered at O . If $\angle OAB$ measures 50° and $\angle BCO$ measures 60° , what is the measure of $\angle AOB$?



8) If $x = \frac{y + a^2}{y - 4a}$, then what is the value of y in terms of a and x ?

9) If $f(x) = 2x^2 - x + 1$, then find the equation tangent to the line to the graph of f at the point $(1,2)$ expressed as $y = mx + b$.

10) If the term in a geometric sequence is 3 and the third term is 48, then what is the fifth term?

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11) Find real numbers A and B such that $x \neq \pm 1$.

$$\frac{2x}{x^2 - 1} = \frac{A}{x-1} + \frac{B}{x+1}$$

12) If $f(x) = \frac{x}{x+3}$, then what is the domain of $f \circ f$?

13) Represent $0.\overline{373} = 0.373373373373 \dots$ as a ratio of two positive integers.

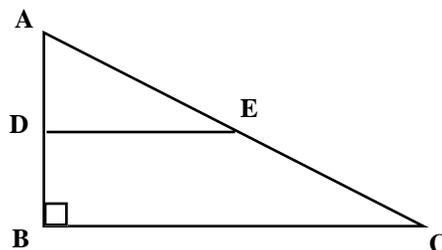
14) Simplify the following expression and express in terms of positive exponents.

$$\frac{15r^{-8}s^4t^{-3}}{5r^{-6}s^4t^{-5}}$$

15) If $F'(x) = \tan x \sec^2 x - x$ and $F(0) = 1$, find $F(x)$.

16) 20 identical balls are to be placed into three urns, labeled A , B , and C . How many ways can the balls be placed so that each urn contains *at least* two balls?

17) In the right triangle $\triangle ABC$, assume $\overline{DE} \parallel \overline{BC}$, $m(\overline{AD}) = 6$, $m(\overline{BD}) = 4$, and $m(\overline{DE}) = 4$. Find $m(\overline{BC})$.



18) Find the equation of the line passing through the point $(2,1)$ and *perpendicular* to the line $3y - x + 15 = 0$.

19) A ball rebounds to half the height from which it is dropped. If it is dropped from 10 feet, how far does it travel from the moment it is dropped until the moment of its eighth bounce?

20) How many four-digit, *even* integers can be formed from the digits 1, 2, 4, 8, 9 if repetition of the digits is allowed?

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(Section II)

1) If the line $y = 8x - 24$ intersects the line $y = mx + 12$ in the *fourth* quadrant, what are the possible values of m ?

2) If $f(x) = x^2 + x$ for all x and $f(a - 1) = \frac{1}{4}$, what is the value of a ?

3) If $f(x) = -x^3 - 2x$, compute and simplify (assume $h \neq 0$):

$$\frac{f(x+h) - f(x)}{h}$$

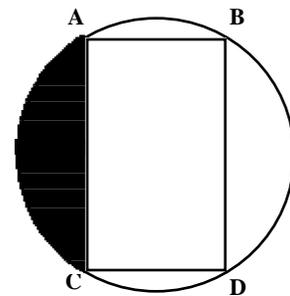
4) Evaluate $\lim_{\theta \rightarrow 0^+} \frac{\sin 8\theta}{3\theta}$.

5) Find the sum of all odd integers for 1 to 199.

6) What is the domain of the function $f(x) = \sqrt{(x-1)(x+2)(x-3)}$?

7) What is the sum of the positive whole numbers that are solutions to $-3n + 3 > -11$?

8) In the figure shown, $ABCD$ is an inscribed rectangle. If the radius of the circle is 1 and $AB = 1$, what is the area of the shaded region?



9) What two digit number is three times the sum of its digits?

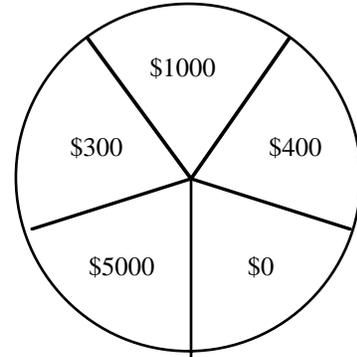
10) A bag contains 6 red marbles and 4 green marbles. Joe reaches into the bag and draws out two marbles at the same time. What is the probability that they are both red?

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11) An operation # is defined for all real numbers a and b by the rule $a \# b = \frac{a^2}{2} + \frac{b}{3}$. If

$-3 \# x = 0$, what is x ?

12) The following spinner is given. Each region has the same area. What is the probability that you will earn exactly \$1700 in your first three spins?



13) Define a function that expresses the area A of a square as a function of the length d of a diagonal of the square.

14) Find a pair of real numbers c and d whose difference is 5 and whose product is as *small* as possible.

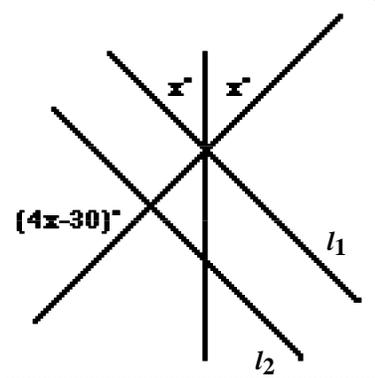
15) What is the coefficient of x^8y^4 in the expansion of $(3x^2 + 2y)^8$?

16) The statement: “If x is a member of set S , then x is not a member of set T ” is logically equivalent to

- I) If x is a member of set T , then x is not a member of set S .
- II) If x is not a member of set T , then x is a member of set S .
- III) If x is not a member of set S , then x is a member of set T .

- a) I only b) II only c) III only
- d) I and II e) II and III

17) In the figure shown, if $l_1 \parallel l_2$, then find the value of x .



18) If $\cos x = 0.5$ and $-\frac{\pi}{2} < x < 0$, then find $\sin 2x$.

19) If $f(x) = \frac{1}{2x+1}$, then find the inverse function $f^{-1}(x)$.

20) Find all values of x which satisfy the inequality

$$\frac{2x-5}{x+6} \leq 1.$$