LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2002 Senior Examination
(Section I)

NAME: ______________________________ Address: ______________________________

SCHOOL: ______________________________ City/ZIP: ______________________________

Telephone: ______________________________

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the volume of a sphere with radius 8 m. 1) ______________________ m^3

2) Convert $0.\overline{345} = 0.345345...$ into a fraction expressed in lowest terms. 2) ______________________

3) Assume $f(x) = 2x^3$. Evaluate and simplify $\dfrac{f(x+h) - f(x)}{h}$. 3) ______________________

4) How many ways can 25 indistinguishable balls be placed into 3 distinguishable urns if each urn must be nonempty? 4) ______________________

5) Find $\displaystyle\lim_{x \to 2} \frac{2x^2 - 6x + 4}{x^2 + 3x - 10}$. 5) ______________________

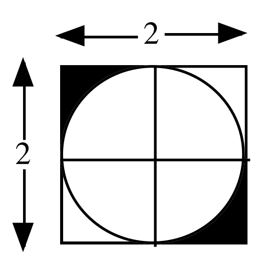
6) If $\sin x = \frac{2}{3}$ and $\sec y = \frac{4}{3}$ where $0 < x < \frac{\pi}{2}$ and $0 < y < \frac{\pi}{2}$, evaluate $\sin(x + y)$. 6) ______________________

7) Find all real numbers x satisfying 7) ______________________
$$|x + 2|^2 + 2|x + 6| - 16 = 0$$

8) Find the domain of the function $f(x) = \dfrac{\sqrt{x+7}}{x-4}$. 8) ______________________

9) What is the smallest number of marbles that can be divided equally among 8 boys, then among 9 boys, then among 12 boys and finally among 15 boys? 9) ______________________

10) Consider the diagram below. What is the area of the shaded region? 10) ______________________



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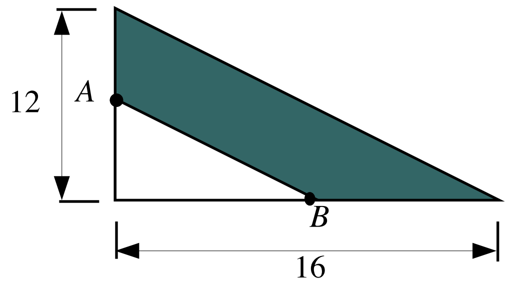
11) The shaded region below is that of a trapezoid. Determine the height of the trapezoid if A and B below are midpoints. 11) ____________



12) Find all real numbers x satisfying $x^3 - 5x^2 + 8x = 4$. 12) ____________

13) Find all real numbers x satisfying $x^{\ln x} = e^2 x$. 13) ____________

14) Compute $(2^0 + 2^1 + \ldots + 2^{11}) - (2^0 + 2^2 + \ldots + 2^{10})$. 14) ____________

15) Assume $\frac{p}{q}$ is a positive rational number in lowest terms. 15) ____________

List all pairs (p, q) such that $9\left(\frac{q}{p}\right) = \frac{p}{q}$.

16) Find the sum of the integers 21 through 74 inclusive. 16) ____________

17) If $\ln x = A$ and $\ln y = B$, then write the following in the form $kA + lB$ where k and l are rational numbers 17) ____________

$$\ln\left(\sqrt[10]{x^3y^4}\right)$$

18) Solve $\frac{1}{x+2} \geq \frac{2}{5}$. 18) ____________

19) Find the equation, in slope-intercept form, of the line which passes through the point $(1, 2)$ and is parallel to the line with equation $10x = 5y + 20$. 19) ____________

20) How many integer triples (x, y, z) satisfy 20) ____________

$x^2 + y^2 - 4z - 3 = 0$?

A) 0 B) 1 C) infinitely many D) none of the above

LUZERNE COUNTY MATHEMATICS CONTEST

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Wilkes University - 2002 Senior Examination
(Section II)

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Solve for x: $8^{3x} = 5\,sin\,(\frac{\pi}{2}) + 6\,cos\,(\frac{\pi}{3})$ 1) ______________________

2) Assume a rectangle has an area 60 m^2 and a diagonal of length 13 m. Find the dimensions of the rectangle. 2) ______m. by______m.______

3) What is the probability of rolling a sum of 5 or 8 on two fair dice? 3) ______________________

4) Find the perimeter of the closed region bounded by the x-axis, the y-axis, the line $x = 3$ and the line $\frac{4}{3}x + y - 7 = 0$. 4) ______________________

5) Find all the real roots of $p(x) = x^3 + x^2 + 9x + 9$. 5) ______________________

6) Express the complex number $-4\sqrt{3} + 4i$ in the form $r(\,cos\,\theta + i\,sin\,\theta\,)$ where $r > 0$ and $0 \le \theta \le 2\pi$. 6) ______________________

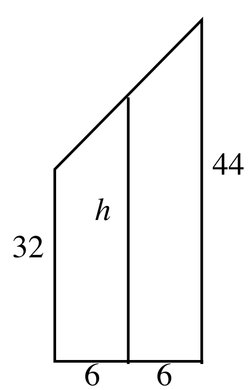
7) Find $\lim_{x \to \infty} \dfrac{5\,sin\,x + cos\,x}{x}$. 7) ______________________

8) Compute $sin\,(\frac{\pi}{12})$. 8) ______________________

9) Which number best completes the following sequence? 9) ______________________
7, 19, 9, 18, 12, 18, 16, 19, ___

A) 17 B) 21 C) 12 D) 19

10) What is the value of h in the trapezoid below? 10) ______________________



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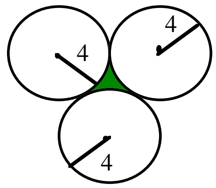
11) Three circles, each having a radius of 4 units are mutually tangent. Find the area of the shaded area between the circles. 11) ______________________



12) Find the constant term in the expansion of $(y - \frac{1}{y})^{10}$. 12) ______________________

13) Compute $\lim_{x \to 1} \frac{5x^2 - 15x + 10}{x^2 - 4x + 3}$. 13) ______________________

14) Assume a person flips five fair coins. What is the probability of obtaining at least 4 heads? 14) ______________________

15) Compute $log_2 (log_3 (9^8))$. 15) ______________________

16) Find all real numbers k so that $2 - \sqrt{3}$ is a root of 16) ______________________

$p(x) = x^2 - 4x + k$

17) Find the ordered pair (x, y) which lies on the line with equation $y = 3x + 10$ and the line with equation $y = 5x - 4$. 17) $x =$ ________ $y =$ ______

18) In a certain arcade a blue token is worth 5 yellow tokens; a yellow token is worth one-fourth of a green token; and a red token is worth one-half of a yellow token. If a blue token is worth 10 points, how much are 6 red tokens, 3 yellow tokens and 2 green tokens worth? 18) ____________________ points

19) An amoeba propagates by simple division. Suppose each split takes 4 minutes to complete. When such an amoeba is placed in a glass container, the container is full of amoebas in one hour. How long would it take for the container to be filled if we start with 8 amoebas instead of only 1? 19) ____________________ minutes

20) Assume a sequence is recursively defined as follows: 20) ______________________

i) $a_0 = 5$

ii) $a_n = a_{n-1} + n$ for any $n \geq 1$.

Calculate a_{100}.