# LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2003 Senior Examination
(Section I)

**NAME:** _____ Address: _____

**SCHOOL:** _____ City/ZIP: _____

Telephone: _____

**Directions:** For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

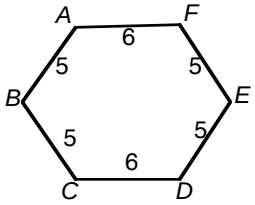
1) What is the smallest positive integer divisible by both 42 and 80? 1)_____

2) What is one third of one quarter of one half of 192? 2) _____

3) Find the perimeter of the triangle formed by the coordinate axes and the line $3x + 5y - 15 = 0$. 3) _____

4) Find the equation of the circle with center (3, -7) that is tangent to the $y$-axis. 4) _____

5) If $\overline{AC} = \overline{FD} = 8$ and $\overline{BE} = 12$, find the area of the hexagon below. 5) _____




6) What is the minimum value of the function $f(x) = 2x^2 - 12x + 16$ ? 6) _____

7) Find all real numbers $x$ that satisfy the following: 7) _____

$$\left| \frac{x - 3}{x + 4} \right| < 2$$

8) Assume that you drop four marbles numbered 1, 4, 6, and 11 into a bin. Assume you draw a pair of marbles If the sum is 17, you win. If not, you put the marbles back into the bin and re-draw. What is the probability you win in 2 or fewer selections? 8) _____

9) Find $\lim_{x \to 0^+} \frac{[x]}{x}$, where $[x]$ denotes the greatest integer less than or equal to $x$. 9) _____

10) Solve for $x$ : $e^{\sqrt{x}} + \frac{14}{e^{\sqrt{x}}} = 9$ 10) _____

**(OVER)**

11) How many ways are there to place 10 indistinguishable balls into 3 different urns if each urn contains at least one ball? 11) _____

12) $\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x =$ 12) _____

A) $\sin (x + \frac{7\pi}{6})$

B) $\sin (x + \frac{11\pi}{6})$

C) $\sin (x - \frac{5\pi}{6})$

D) both A and C

E) none of the above

13) If $\tan \theta = \frac{7}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find $\sin \theta$ . 13) _____

14) A combination lock shows the numbers from 1 through 25. If an individual decides to try all possible three-number orderings, and can try 5 three-number orderings, per minute, what is the probability that the lock will be opened after 4 hours? 14) _____

15) Given the following polar equation, convert it to a rectangular equation describing $y$ in terms of $x$. 15) $y =$ _____

$$r = \frac{1}{1 + \sin \theta}$$

16) Express $\frac{(x + \frac{1}{y})^3 (x - \frac{1}{y})^4}{(y - \frac{1}{x})^4 (y + \frac{1}{x})^3}$ in the form $(\frac{x}{y})^n$, where $n$ is a positive integer. 16) _____

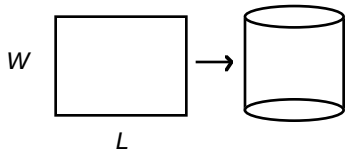
17) Given $a_0 = 1$, and $a_n = \frac{2a_{n-1}}{3}$ for all $n \geq 1$. 17) _____

Find $a_0 + a_1 + ... + a_5$ .

18) Find all real solutions to the equation $x^{10} - 6x^5 + 2 = 0$. 18) _____

19) An architect designs a theater with 10 seats in the first row. Each subsequent row will have four more seats than the previous row. If the theater is to have a seating capacity of 960, how many rows must the architect use in his design? 19) _____

20) A rectangular piece of cardboard is to be rolled into a right circular cylinder with open ends and having a volume of 400 $cm^3$. The area of the cardboard is 600 $cm^2$. What are the length ($L$) and width ($W$) of the cardboard? 20) $L =$ ____ cm ____ $W =$ ____ cm



# LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2003 Senior Examination
(Section II)

**NAME:** _____ Address: _____

**SCHOOL:** _____ City/ZIP: _____

Telephone: _____

**Directions:** For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the volume of a sphere with radius 3 in. 1) _____ $in^3$

2) Find a function of the form $f(x) = A\,e^{Bx}$, if $f(0) = 6$ and $f(3) = 12$. 2) _____

3) One diagonal of a square has endpoints (-4, 3) and (5, -6). Find the endpoints of the other diagonal. 3) _____

4) Two supplementary angles are in a ratio of 5 to 7. Find the measure of each angle (in degrees). 4) _____

5) Find the vertex **and** axis of symmetry of the parabola $x - 17 = 3y^2 + 12y$ . 5) vertex : _____
axis of symmetry : _____

6) Which of the following statements are true? 6) _____

I) Every square is a rhombus
II) Every trapezoid is a parallelogram
III) Every square is a rectangle

A) I only
B) II only
C) III only
D) both II and III
E) both I and III

7) Express $.\overline{54}$ as a common fraction in lowest terms. 7) _____

8) The relation $3x^2 - 18x + 4y^2 - 32y + 91 = 0$ defines. 8) _____

A) an ellipse
B) a parabola
C) two distinct lines
D) a point
E) none of the above

9) Evaluate $\lim_{x \to -2} \dfrac{x^2 + 6x + 8}{x^2 - 5x - 14}$ 9) _____

10) Find all real solutions of $\sqrt[3]{\sqrt{x - 4} - 5} = \sqrt{8}$ 10) _____

**(OVER)**

11) Evaluate $( \log_4 5 )( \log_5 6 )( \log_6 7 ) \cdots ( \log_{255} 256 )$. 11) _____

12) Find all values of $x$ that satisfy $\left| 6 - \frac{3}{x} \right| \leq 2$ ? 12) _____

13) Find real numbers $A$ and $B$ such that 13) $A$=_____ $B$=_____
$\left( \frac{\sqrt{3}}{4} + \frac{1}{4} i \right)^6 = A + Bi.$

14) Solve for $x$ : $\log_{10} ( x - 5 ) = \log_{10} x - \log_{10} 5$ . 14) $x$ = _____

15) Compute $\sin \left( \frac{5\pi}{12} \right)$. 15) _____

16) Find the coefficient of $x^{46}y^2$ in the expansion of $( x^2 - 5y )^{25}$ . 16) _____

17) The exact value of 17)_____
$\cos \frac{\pi}{100} + \cos \frac{2\pi}{100} + \cos \frac{3\pi}{100} + \ldots + \cos \frac{199\pi}{100}$ is

A) 1
B) -1
C) 0
D) $\frac{\sqrt{2}}{2}$
E) none of the above

18) Find the polynomial with real coefficients of the smallest possible degree for which $i$ and $5 + i$ are zeros and in which the coefficient of the highest power of $x$ is 1. 18)_____

19) Suppose two pumps are used to empty a large tank. It takes 14 hours to empty the tank using both pumps. It is known that one pump is 10% faster than the other. How much time (in hours) would be needed to empty the tank if only the faster pump was used? 19)_____ hours

20) A rectangle is inscribed in an equilateral triangle with a perimeter of 24 cm (see the figure below). Express the area, $A$, of the shaded region as a function of $x$. 20) $A(x)$ = _____