LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2004 Senior Examination
(Section I)

NAME: _____ Address: _____

SCHOOL: _____ City/ZIP: _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Which of the following expressions is equivalent to $\dfrac{a^{-2}(b^2c^3)^{-3}}{(a^{-5}b^{-4})^2 c^{-7}}$? 1)_____

A) $a^8b^2c^2$ B) $\dfrac{a^8c^2}{b^2}$ C) $\dfrac{a^8b^2}{c^2}$ D) $\dfrac{b^8c^2}{a^2}$

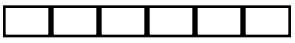
E) None of the above

2) If θ is an acute angle and $\sin\theta = \frac{5}{13}$, what is $\cos\theta$? 2) _____

3) Find all values x such that $x(x - 1) = 1$. 3) $x =$ _____

4) What is the sum of the interior angles of an octagon? Express your answer in radians. 4) _____

5) How many distinct rectangles are in the diagram below? 5) _____



A) 6 B) 12 C) 18 D) 24 E) None of the previous choices

6) A star basketball player makes 15 of 20 free throws to start the season. The player wishes to have made at least 85% of attempted free throws by the end of the season. If the player attempts 130 more free throws during the season, what is the minimum number that must be made for the goal to be achieved? 6) _____

7) Find A, B, and C: $\dfrac{5x^2 - 29}{x^3 + 4x^2 + x - 6} = \dfrac{A}{x+2} + \dfrac{B}{x-1} + \dfrac{C}{x+3}$ 7) $A=$ _____ $B=$ _____ $C=$ _____

8) Solve: $\log_3(x + 2) + \log_3(x - 6) = 2$. 8) $x =$ _____

9) Solve the equation $e^{2x} + 5e^x - 2 = 0$ 9) $x =$ _____

10) Evaluate: $\dfrac{1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots}{1 + 4 + 7 + \ldots + 49}$ 10) _____

(OVER)

11) Given $f(x) = \sqrt{x}$ and $h \neq 0$, $\dfrac{f(x+h) - f(x)}{h}$ is equal to 11) _____

A) $\dfrac{1}{2\sqrt{x}}$ B) $\dfrac{1}{\sqrt{x+h} + \sqrt{x}}$ C) $\dfrac{2}{\sqrt{x+h} + \sqrt{x}}$

D) $\dfrac{1}{\sqrt{x}}$ E) None of the previous choices

12) Find a polynomial function f of degree 3, such that $f(5) = 9$ and the zeroes of $f(x)$ are 0, 2, and 6. 12) $f(x)$=_____

13) For what value of θ does $f(\theta) = 4 \cos 3\theta$ attain its minimum on $[-\frac{\pi}{3}, 0]$? 13) _____

$f^{-1}(x)$=

14) Consider the function $f(x) = 2x^2 + 1$, for $x < 0$. Find $f^{-1}(x)$ and its domain. 14) _____ dom(f) = _____

15) Through how many radians does the hour hand on a clock move between 2:00 pm and 7:15 pm on the same day? 15) _____

16) sin $3x$ equals 16) _____

A) sin $2x$ cos x + cos $2x$ sin x C) sin $2x$ + cos $2x$ + cos x + sin x

B) sin $2x$ cos x - cos $2x$ sin x D) Both A and C

17) An isosceles triangle has a perimeter of 12 cm. Express the area of the triangle as a function of the length B of the base of the triangle. 17) _____ cm

18) Find the area of the region that lies outside the circle $x^2 + y^2 = 9$ but inside the circle $x^2 + y^2 - 6y - 40 = 0$. 18) _____

19) The ratio of the units digit to the tens digit of a two digit number is two to one. The number formed by reversing the digits is 6 less than 2 times the original number. What is the original number? 19) _____

20) The locus of points in the plane represented by 20) _____

$-2x^2 - 3y^2 + 6x + 7y + 6 = 0$ is

A) a circle
B) a non-circular ellipse
C) an hyperbola
D) a point
E) none of the previous choices

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(Section II)

NAME: _____ Address: _____

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the surface area of a sphere with diameter 4 units. 1)_____

2) What value of A makes the line $Ax + 5y = 25$ perpendicular to line $25x = 35y - 70$? 2) $A=$_____

3) Evaluate $[-\pi] + [7] + [\frac{8}{5}]$ where $[x]$ is the greatest integer less than or equal to x. 3)_____

4) The cost of a pizza is proportional to its area. If a pizza with a 14 inch diameter costs \$7.00, how much does a pizza with a 16 inch diameter cost? (Express your answer to the nearest cent) 4) \$_____

5) How many distinct numbers of length 5 can be represented using five octal digits? 5) _____

6) Consider the following point $P = (16, \frac{7\pi}{6})$ in polar coordinates. Convert this to a point in rectangular coordinates. 6) _____

7) $(-1 + \sqrt{3}\, i)^{12} =$ 7) _____

A) -1 B) 1 C) 512 D) -512 E) 4096 F) -4096

8) Suppose that $N = 2^6 \cdot 3^3 \cdot 5^4$. How many divisors of N are multiples of 6? 8) _____

9) What is the smallest positive integer that is a product of four distinct odd primes? 9) _____

10) Find all real solutions to the equation $5\sqrt[4]{x} = 14 - \sqrt{x}$. 10) $x=$_____

(OVER)

11) The side of a hill makes a 30º angle with the horizontal. If you climb 1,000 feet up the hill, how far will you rise vertically?

11) _____ft____

$x=$

12) Solve the following linear system for x and y in terms of K and L.

$$3x + 4y = K^2$$
$$6x + 3y = L - K$$

12) __y=_____

13) $\lim_{x \to 0} \frac{[x]}{x}$ is equal to

A) 1 B) 0 C) -1 D) Does not exist

where $[x]$ is the greatest integer less than or equal to x.

13) _____

14) How many real solutions does $3^{x^2 - 2x + 1} = 1$ possess?

14) _____

15) If $m = \frac{ab}{a - b}$, find b in terms of a and m, given that $a \neq -m$.

15) __b=_____

16) Find the inverse, $f^{-1}(x)$, of the function $f(x) = \ln(\log_7 x)$.

16) __$f^{-1}(x)$=_____

17) $h(x) = x^3(x^2 + 1)^2$ is

A) an odd function
B) an even function
C) neither an odd nor even function

17)_____

18) An equilateral triangle has a height of 10 inches. How long is one of its sides?

18)_____in__

19) An airplane flew with a constant wind for 3.5 hours and returned the same distance against the same wind in 4.5 hours. If the plane's speed was a constant 350 mph, how fast was the wind?

19)_____mph_

20) If $(r + \frac{1}{r})^2 = 3$, what is $r + \frac{1}{r^3}$?

20) _____