LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2005 Senior Examination
(Section I)

NAME: _____ **Address:** _____

SCHOOL: _____ **City/ZIP:** _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

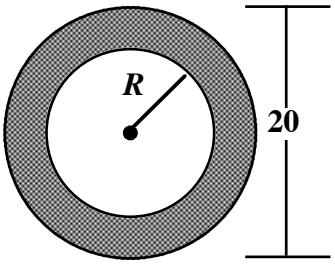
1) What is the diameter of a sphere whose surface area is 100π ? **1)**_____

2) Suppose the slope of the line segment connecting the points $(x+2,4)$ and $(3,7)$ is $\frac{9}{4}$. What is x? **2)** $x =$_____

3) If $f(x)=\frac{5+x}{2}$, find $f^{-1}(3)$ where f^{-1} denotes the inverse function of f. **3)**_____

4) Evaluate $\sin(\pi xy)+x^2+y^2$ when $x = 2$ and $y = 7$. **4)**_____

5) Find the value of R if the area of the shaded region is 64π cm^2 . **5)** $R =$_____cm.



6) What is the perimeter of the triangle whose vertices are (-3,-4), (1,0), and (2,1) ? **6)**_____

7) Find the period of $y = 3\sin(2x-\pi)$. **7)**_____

8) A student taking an exam is required to answer 16 out of 20 questions. In how many ways can the 16 questions be selected if exactly 3 of the first 4 questions must be answered? **8)**_____

9) Find all solutions to the equation $\frac{1}{2}\sec x = \tan x$ for x in $[0, 2\pi]$. **9)** $x =$_____

10) How many distinct subsets of size 2 does the set {1,2,3,4,5} possess? **10)**_____

(a) 5 **(b)** 10 **(c)** 20 **(d)** 120

(OVER)

11) Find all real solutions of $e^{2x} - 7e^{x} + 10 = 0$. **11)** $x =$_____

12) Given $f(x) = 6x^2$ and $h \neq 0$, compute and simplify $\dfrac{f(x+h) - f(x)}{h}$. **12)**_____

(a) 0 **(b)** $12x$ **(c)** $12h+6x$
(d) none of the above

13) Find a polynomial of degree 2 with integer coefficients, and leading coefficient 1, whose distinct roots are $2 \pm 3i$. **13)**_____

14) How many distinct solutions does the equation $2\sin 3\theta = 1$ have on $[0, \pi]$? **14)**_____

(a) 1 **(b)** 2 **(c)** 3
(d) none of the above

15) If $f(x) = \dfrac{x^2 - 36}{x - 6}$ and $x \neq 6$, how must $f(6)$ be defined so f is continuous on $(-\infty, \infty)$? **15)**_____

16) Express the following as a polynomial in x: $\cos(2\arccos x)$. **16)**_____

17) The set of all (x, y) for which $(x, y) = (-x, y)$ is represented by **17)**_____

(a) the line $y = x$ **(b)** the line $y = -x$
(c) the x-axis **(d)** the y-axis

18) Terry paints twice as fast as Sue and three times as fast as Ken. If it takes them 90 minutes to paint a room with all three working together, how long would it take Ken if he works alone? **18)**_____ min.

19) The equation(s) of the vertical asymptotes of the graph of **19)**_____

$f(x) = \dfrac{x}{x^2 + 5}$ is/are

(a) $x = 0$ **(b)** $x = \pm 3$
(c) $y = \pm 3$ **(d)** the graph of f does not have any vertical asymptotes

20) If $f(x) = 2x - 1$, find $f^8(2x - 1)$ where **20)**_____

$f^8(x) = f(f(f(f(f(f(f(f(x)))))))$.

LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2005 Senior Examination
(Section II)

NAME: _____ **Address:** _____

SCHOOL: _____ **City/ZIP:** _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the slope of the line perpendicular to the line $5x - 4y = 12$. **1)**_____

2) Assume we roll two four-sided dice. The faces are numbered 1,2,3, and 4, respectively. What is the probability of observing a sum of exactly five on the reckoned faces? **2)**_____

3) An equilateral triangle has a height of 25 inches. How long is one of its sides? **3)**_____ in.

4) Solve the following linear system of equations for x and y. **4)** $x =$_____ $y =$_____

$2x - 6y = 10$

$4x + 3y = 5$

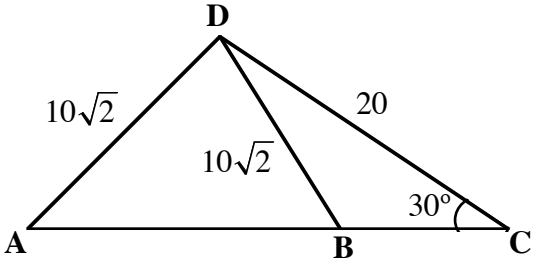
5) Find $\dfrac{f(x+h)-f(x)}{h}$ where $f(x) = \dfrac{1}{x^2+1}$. **5)**_____

6) Find the inverse, $g(x)$, of the function $f(x) = \dfrac{4^x}{1+4^x}$. **6)** $g(x) =$_____

7) Find the smallest number that is a product of five distinct primes. **7)**_____

8) Find all real solutions to the following equation : $\left|\dfrac{3}{7}x - \dfrac{1}{2}\right| = 1$. **8)** $x =$_____

9) Given the figure below, find $\angle BDC$ (in degrees). **9)**_____ degrees



10) Find the one real solution of $x \ln x = 3e^3$. **10)** $x =$_____

(OVER)

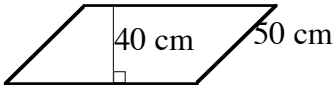
11) $\lim_{h\to 0} \dfrac{(3+h)^3 - 27}{h} =$ **11)**_____

(a) 0 **(b)** 1 **(c)** 9 **(d)** 27

(e) positive infinity

12) Assume $N = 2^7 \cdot 3^4 \cdot 5^2$. How many divisors of N are multiples of 18? **12)**_____

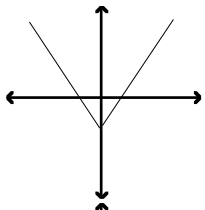
13) The parallelogram below has an area of 3200 cm^2. What is its perimeter? **13)**_____cm.



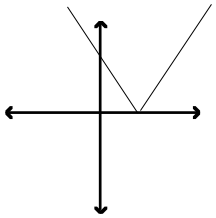
14) Solve for x: $\log_{10}(x+2) - \log_{10}(2x+3) = 2$. **14)**_____

15) Which of the following graphs most closely resembles the graph of $f(x) = \big|\,|x| - 3\,\big|$? **15)**_____

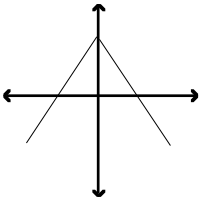
a)



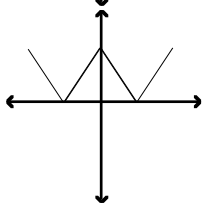
b)



c)



d)



16) Find A, B, and C if $\dfrac{2x^2 - 4x - 4}{x^3 - 2x^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-2}$ is an identity. **16)** $A =$_____ $B =$_____ $C =$_____

17) Find a function, $f(x)$, whose graph is a parabola that passes through the points (1, 3), (-1, -5) and (2, 10). **17)** $f(x) =$_____

18) Find $\lim_{x\to 2} \dfrac{x^2 - 3x + 2}{x^2 - 5x + 6}$. **18)**_____

19) A *google* is 10^{100} and a *googleplex* is 10^{google}. Find $10 \log(\log(\text{googleplex}))$. **19)**_____

20) How many ways are there to place 11 indistinguishable balls into 3 distinct urns so that each urn contains at least one ball? **20)**_____