LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2006 Senior Examination
(Section I)

NAME: _____ **Address:** _____

SCHOOL: _____ **City/ZIP:** _____

Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the length of the unique line segment connecting the points (0, 2) and (4, 7)? **1)**_____

2) How many distinct primes are in the prime factorization of 243? **2)**_____

3) The slope of the line perpendicular to the line $x - 2y = 7$ is **3)**_____

(a) $\frac{1}{2}$ **(b)** $-\frac{1}{2}$ **(c)** -2 **(d)** 2
(e) none of the above

4) Suppose $f(x) = 2x^2 + 3$. Evaluate $f(2x) - 2f(x)$. **4)**_____

5) An isosceles right triangle has a hypotenuse of length 10. How long is the length of either of the triangles legs? **5)**_____

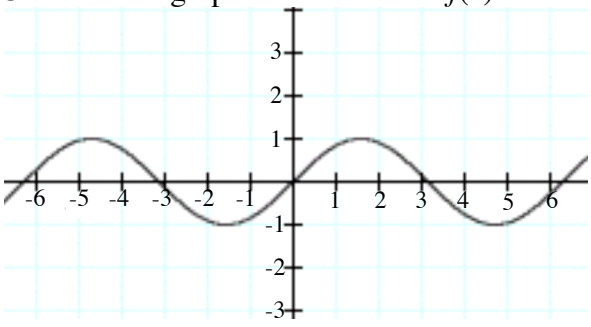
6) If $T = 2\pi\sqrt{\frac{m}{g}}$ for $m > 0$, $g > 0$, solve for g in terms of T and m. **6)** $g =$_____

7) If $f(x) = \frac{x-1}{x^2+4x-5}$, how must $f(1)$ be defined so f is continuous on $\{x | x \neq -5\}$. **7)** $f(1) =$_____

8) If $f(x) = \log\left(\frac{x+1}{x-1}\right)$, find all x such that $f(x) = f(2) + f(3)$. **8)** $x =$_____

9) Find all real solutions x to $\sqrt{2x-3} - \sqrt{x+7} = 2$. **9)** $x =$_____

10) Consider the graph of the function $f(x)$ below. This function **10)**_____



(a) is an even function **(b)** is an odd function **(c)** is **not** periodic
(d) each of **a** - **c** is true **(e)** none of **a** - **c** are true

(OVER)

11) Find the axis of symmetry for the parabola $x = y^2 - 8y + 20$. **11)**_____

12) Assume the population of fish in a given lake at time t months is given by the function $P(t) = \dfrac{50{,}000}{2 + 25e^{-\frac{t}{5}}}$, $t \geq 0$. What is the upper limit of the fish population? **12)**_____

13) How many gallons of a 10% brine solution and a 40% brine solution must be mixed to obtain 20 gallons of a 25% brine solution? **13)**_____

14) Find the sum of the integers from 101 to 200 inclusive. **14)**_____

15) The graph of the function $f(x) = mx + b$ is a line. What is the y-intercept of the graph of $(f \circ f \circ f)(x)$? **15)**_____

16) Find a polynomial of degree 3 with integer coefficients and leading coefficient 3 whose distinct roots are 1, $2i$, and $-2i$ respectively. **16)**_____

17) How many solutions does the equation $3\sin\left(\dfrac{1}{x}\right) = -2$ possess on the interval $(0, \pi)$? **17)**_____

(a) 0 **(b)** 1 **(c)** 4 **(d)** 8 **(e)** none of the above

18) A circle has its center on the graph of $y = 2x + 4$ and is tangent to the graph of $y = x + 1$ at the point (6, 7). Find the center of the circle. **18)**_____

19) Find all real numbers x such that $\left(\dfrac{2}{\sqrt{\pi}}\right)^x = \text{Arc}\sin\left(\dfrac{\sqrt{2}}{2}\right)$. **19)** $x =$_____

20) Assume $N = 2^5 \cdot 7^{11} \cdot 11^7$. How many divisors of N are multiples of 22? **20)**_____

LUZERNE COUNTY MATHEMATICS CONTEST

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(Section II)

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Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Express $\log_5\left(\frac{1}{25}\right)$ as a rational number. **1)**_____

2) Assume the slope of the line segment containing the points $(x+1, 4)$ and $(2, x+5)$ is 2. What is x ? **2)** $x =$_____

3) Find the smallest prime number greater than or equal to 80. **3)**_____

4) Charles owns 10 different suits. He is going on a business trip. and needs to pack 5 suits. How many different ways can he select the suits? **4)**_____

5) Find all real solutions to $x^4 - 4x^2 + 4 = 0$. **5)** $x =$_____

6) Express $\sec\left(-\frac{\pi}{3}\right)$ as a rational number. **6)**_____

7) Find the range of the function $f(x) = -2x^2 + 5x + 8$. **7)**_____

8) Which of the following is the negation of the statement? **8)**_____

Nancy always wears a red sweater on Friday.

(a) Nancy never wears a red sweater on Friday.
(b) There exists a Friday on which Nancy does not wear a red sweater.
(c) Nancy always wears a red sweater on days other than Friday.
(d) None of the above.

9) If $\log(a) = K$ and $\log(b) = L$, express $\log((a^2b^5)^3)$ as a sum of two terms involving K and L. **9)**_____

10) Find $\frac{f(x+h) - f(x)}{h}$ where $f(x) = x^3$. **10)**_____

(OVER)

11) Find all values of x that satisfy the following nonlinear system of equations. **11)** $x =$ _____

$$x + y = -3$$
$$x^2 + y^2 = 17$$

12) The equation $x^2 - 6x + 2y^2 - 16y + 42 = 0$ describes a(n) **12)** _____

(a) circle **(b)** parabola **(c)** hyperbola
(d) line **(e)** none of the above

13) If $\sin y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$, compute $\sec y$. **13)** _____

14) If a and b are non-zero rational numbers and $\frac{a^2}{b^2} < 3$, then **14)** _____

(a) $\frac{(a+3b)^2}{(a+b)^2} < 3$ **(b)** $\frac{(a+3b)^2}{(a+b)^2} > 3$

(c) $\frac{(a+3b)^2}{(a+b)^2} = 3$ **(d)** $\frac{(a+3b)^2}{(a+b)^2} \geq 3$

(e) none of **a** - **d** are true for rational numbers a and b

15) Assume a deck of twelve cards contains 3 identical two's, 3 identical three's, 3 identical four's, and 3 identical five's. How many 4 card hands are possible? **15)** _____

16) How many real solutions does the equation $x^3 - 5x^2 + 8x - 4 = 0$ possess? **16)** _____

17) Consider a triangle ABC where $\left|\overline{AB}\right| = 10$, $\left|\overline{BC}\right| = 16$, and $\angle ABC = \frac{2\pi}{3}$ radians. Find $\left|\overline{AC}\right|$. **17)** _____

18) The shortest side of a right triangle has length 3. Find the shortest side of a similar right triangle whose area is twice that of the original triangle. **18)** _____

19) Find the solution set to the inequality $|\sin x| < \frac{1}{2}$ on $[0, 2\pi]$. **19)** _____

20) What is the probability of rolling two fair six-sided dice and obtaining a sum that is divisible by 5? **20)** _____