LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics
Wilkes University - 2007 Senior Examination
(Section I)

NAME: _____ **Address:** _____

SCHOOL: _____ **City/ZIP:** _____

Telephone: _____

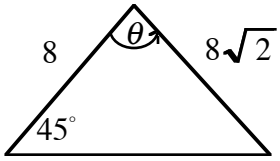
Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) Find the distance between $P = (0, 3)$ and $Q = (7, 8)$. **1)**_____

2) Express the following as an integer: $(\log_5 3)(\log_3 125)$. **2)**_____

3) Find the remainder when $7x^{1200} + 31x^{755} - 8x^9 + 12$ is divided by $x + 1$. **3)**_____

4) Find the value of θ in the following triangle:



4) $\theta =$_____

5) If $f(n + 1) = f(n) + 4$ and $f(1) = 2$, then $f(100) = ?$ **5)**_____

6) Assume a water tank is in the shape of a right circular cone with the the vertex at the bottom. The height of the tank is 20 feet and the radius of the tank is 12 feet. The tank contains 15π cubic feet of water. What is the height of the water in the tank? **6)**_____ feet

7) If the equation $\cos^2 x - \sin x + a = 0$ has a solution in $(0, \frac{\pi}{2}]$, then a will satisfy **7)**_____

(a) $-1 \le a \le 1$ **(b)** $-1 < a \le 1$ **(c)** $-1 < a < 0$ **(d)** $a \le -\frac{5}{4}$

8) Find all real solutions of $\frac{18}{3 + e^{-x}} = 3$. **8)** $x =$_____

9) What is the radius of the circle $x^2 + y^2 + 6x - 10y - 17 = 0$? **9)**_____

10) How many distinct two-element subsets does the set $\{ A, B, C, D, E, F \}$ possess? **10)**_____

(OVER)

11) Suppose that a box contains 4 identical white balls and 5 identical black balls. If a ball is randomly selected, what is the probability that the ball selected is white? **11)**_____

12) If $25^x = 2000$ and $80^y = 2000$, then $\frac{1}{x} + \frac{1}{y}$ equals ? **12)**_____

(a) 2 **(b)** 1 **(c)** $\frac{1}{2}$ **(d)** $\frac{3}{2}$

13) Find a function whose graph is a vertical parabola that passes through the points (2, 10), (0, -2), and (-1, -5). **13)**_____

14) Find the inverse $g(x)$ of the function $f(x) = 3\log_2 x$ **14)** $g(x)$=_____

15) Let $f(x)$ be an odd function in $\mathbb{R}$ and $g(x)$ be an even function in $\mathbb{R}$. If $f(x) - g(x) = x^2 + 2x + 3$, what is $f(x) + g(x)$? **15)**_____

16) How many integers between 1 and 287 inclusive are divisible by 6? **16)**_____

17) If $x^2 + mx + 36$ is a perfect square trinomial for all real numbers x, then m = ? **17)** m =_____

18) For $y \neq 2x$, $\frac{2x}{2x - y} + \frac{y}{y - 2x}$ equals **18)**_____

(a) 1 **(b)** -1 **(c)** $2x + y$ **(d)** $x + y$

19) Find all solutions to $\csc x = \cot x$ for $x \in [0, 2\pi]$. **19)**_____

20) $\lim_{h \to 0} \frac{2^{2+h} - 4}{h} =$ **20)**_____

(a) 0 **(b)** 4

(c) 4 ln 2 **(d)** none of the above

LUZERNE COUNTY MATHEMATICS CONTEST

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Wilkes University - 2007 Senior Examination
(Section II)

NAME: _____ **Address:** _____

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Telephone: _____

Directions: For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the volume of a sphere whose diameter is 6 units? **1)**_____

2) If an electronics store buys television sets for $480 and wishes to sell the television sets for 25% more than they paid for them, how much should the store charge? **2)**_____

3) If $f(x) = x^2$ and $g(x) = 3x - 1$, what is the product of $(f \circ g)(x)$ and $(g \circ f)(x)$? **3)**_____

4) If $x < 2$, then $\dfrac{x-2}{|x-2|} =$? **4)**_____

5) Two sides of a triangle have lengths 2 and 7 respectively and the third side has a length which is an even integer, then the perimeter of the triangle is **5)**_____

(a) 15 **(b)** 16 **(c)** 17 **(d)** 15 or 17

6) How many distinct primes are less than or equal to 31? **6)**_____

7) If $\dfrac{a^2-9}{a+3} = 0$, then a equals **7)**_____

(a) 3 **(b)** -3 **(c)** ± 3 **(d)** any real number

8) $1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{2} + \dfrac{1}{2\sqrt{2}} + \ldots = ?$ **8)**_____

9) Find $\lim\limits_{x \to 3} \dfrac{x^2 - 8x + 15}{2x^2 - 6x}$ **9)**_____

10) What is the largest integer value of n such that $2^n < 5{,}000{,}000$? **10)** $n=$_____

(OVER)

11) How many ways are there to make 15 cents in change if pennies, nickels and dimes can be used? **11)**_____

12) A storage silo consists of a cylindrical main section and a hemispherical roof. What is the volume of the silo (including the part inside the roof section) if the silo's cylindrical portion is 40 feet tall and has a radius of 10 feet? **12)**_____cubic feet

13) Convert the following point in Euclidean coordinates to polar coordinates: $P = (3, \sqrt{3})$, where the angle θ is in $[0, \pi]$. **13)**_____

14) What is the units digit of 3963^{447} ? **14)**_____

15) Find the area of a triangle with sides 9, 10, and 11 respectively. **15)**_____

16) Assume an operation $\bigstar$ is defined by $x \bigstar y = \dfrac{x}{x+y}$. For any (x, y) such that $x + y \neq 0$, how many ordered pairs (x, y) satisfy the equation $x \bigstar y = 0$? **16)**_____

(a) 0 **(b)** 1
(c) an infinite number **(d)** none of the above

17) Suppose a and b are real numbers satisfying $ab = 1$. Let **17)**_____

$$M = \frac{1}{1+a} + \frac{1}{1+b} \text{ and } N = \frac{a}{1+a} + \frac{b}{1+b}, \text{ then}$$

(a) $M > N$ **(b)** $M = N$
(c) $M < N$ **(d)** not enough information is given to establish a relationship between M and N

18) If it takes 4 quarts of paint to cover a wall that is 8 feet high and 12 feet wide, how many quarts are needed to cover a wall that is 12 feet high and 18 feet wide? **18)**_____quarts

19) If $x^2 + x - 1 = 0$, then $x^3 + 2x^2 + 2000 = ?$ **19)**_____

20) If x is a positive real number and $\left(x^2 + \dfrac{1}{x^2}\right) = 3$, then **20)**_____

$$x^3 + \frac{1}{x^3} = ?$$