

# LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics

Wilkes University - 2008 Senior Examination

(Section I)

NAME: \_\_\_\_\_

Address: \_\_\_\_\_

SCHOOL: \_\_\_\_\_

City/ZIP: \_\_\_\_\_

Telephone: \_\_\_\_\_

**Directions:** For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

1) What is the area of a square with a diagonal of length 2 units? 1) \_\_\_\_\_

2) Assume a container holds 6 identical red jelly beans and 5 identical blue jelly beans. If we reach into the container and randomly select 3 jelly beans, what is the probability all three are blue? 2) \_\_\_\_\_

3) Completely factor  $2x^3 + 4x^2 + 3x + 6$ . 3) \_\_\_\_\_

4) Find all real numbers  $x$  that satisfy the inequality  $|x - 2| > 2 - x$ . 4) \_\_\_\_\_

5) The maximum value of  $\frac{2}{x^2 - 4x + 7}$  is 5) \_\_\_\_\_

(a) 2      (b)  $\frac{2}{3}$       (c)  $\frac{3}{2}$       (d) 1

6) A square and a circle have equal perimeters. What is the ratio of the area of the circle to the area of the square? 6) \_\_\_\_\_

7) Find a polynomial  $f(x)$  with real coefficients such that  $f(x)$  has degree 2, has roots  $(2 + i)$  and  $(2 - i)$ , and has leading coefficient 3. 7)  $f(x) =$  \_\_\_\_\_

8) If  $4x^2 - 6x + m = (x - 3)(ax + b)$  where  $a$ ,  $b$ , and  $m$  are real numbers, what is the value of  $m$ ? 8)  $m =$  \_\_\_\_\_

9) Compute  $\lim_{x \rightarrow 4} \frac{x^3 - 64}{2x - 8}$ . 9) \_\_\_\_\_

10) Express  $\frac{5}{\sqrt[3]{4}}$  in the form  $\frac{a}{b} \sqrt[3]{2}$  where  $\frac{a}{b}$  is a fraction in lowest terms. 10) \_\_\_\_\_

(OVER)

11) If  $x < 2$ , then  $\sqrt{(x-2)^2} + |3-x|$  is  
(a)  $-1$       (b)  $1$       (c)  $2x-5$       (d)  $5-2x$       11) \_\_\_\_\_

12) Suppose  $a_1 = 3$  and  $a_n = 3a_{n-1} + 2$ , Find  $a_7 - a_5$ .      12) \_\_\_\_\_

13) Find the exact value of  $\sin\left(\frac{\pi}{8}\right)$ .      13) \_\_\_\_\_

14) Solve  $\frac{B^2}{4A} + Ax^2 = -Bx$  for  $x$  in terms of  $A$  and  $B$ .      14) \_\_\_\_\_

15) Determine all real solutions of  $\frac{\ln(2x+3)}{\ln(x+1)} = 2$ .      15) \_\_\_\_\_

16) If  $a$  and  $b$  are positive integers and  $143a + 500b = 2001$ , then what is the value of  $a + b$ ?      16) \_\_\_\_\_

17) The number of digits in the number  $N = 5^8 - 2^{12}$  is  
(a)  $5$       (b)  $6$       (c)  $7$       (d)  $8$       (e)  $9$       17) \_\_\_\_\_

18) Express  $\frac{1}{5+7i}$  in the form  $a + bi$  where  $a$  and  $b$  are rational numbers.      18) \_\_\_\_\_

19) Find all real solutions to the inequality  $\left|\frac{2}{9-x^2}\right| \leq \frac{1}{4}$ .      19) \_\_\_\_\_

20) If  $f(x) = \ln\left(\frac{1+x}{1-x}\right)$  for  $-1 < x < 1$ , then  $f\left(\frac{3x+x^3}{1+3x^2}\right) =$       20) \_\_\_\_\_

- (a)  $-f(x)$       (b)  $2f(x)$       (c)  $3f(x)$   
(d)  $[f(x)]^2$       (e)  $[f(x)]^3 - f(x)$

# LUZERNE COUNTY MATHEMATICS CONTEST

Luzerne County Council of Teachers of Mathematics  
Wilkes University - 2008 Senior Examination  
(Section II)

NAME: \_\_\_\_\_ Address: \_\_\_\_\_  
SCHOOL: \_\_\_\_\_ City/ZIP: \_\_\_\_\_  
Telephone: \_\_\_\_\_

**Directions:** For each problem, write your answer in the space provided. Do not use approximations. Simplify all fractions and radicals. Your answer must be complete to receive credit for a problem.

- 1) What is the volume of a sphere whose diameter is 6 units? 1) \_\_\_\_\_
- 2) What is the largest prime smaller than 50? 2) \_\_\_\_\_
- 3) Find the equation of the line between  $P = (1, 3)$  and  $Q = (2, -7)$ . 3) \_\_\_\_\_
- 4) Compute  $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{2x - 4}$ . 4) \_\_\_\_\_
- 5) If  $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ , then compute  $\frac{5}{x}$ . 5)  $\frac{5}{x} =$  \_\_\_\_\_
- 6) Express  $\frac{\log_4 8}{\log_{\sqrt{5}} 25}$  as a fraction in lowest terms. 6) \_\_\_\_\_
- 7) If  $a, b,$  and  $c$  are positive integers less than 10, then  $(10a + b)(10a + c) = 100a(a + 1) + bc$  if 7) \_\_\_\_\_  
(a)  $b + c = 10$  (b)  $b = c$  (c)  $a + b = 10$   
(d)  $a = b$  (e)  $a + b + c = 10$
- 8) Let  $a$  and  $b$  be positive real numbers. Consider the binary operation 8) \_\_\_\_\_  
 $a \otimes b = \frac{ab}{a + b}$  on the set of positive real numbers. Express  $4 \otimes 4 \otimes 4$   
as a rational number.
- 9) How many distinct strings can be formed by permuting the letters 9) \_\_\_\_\_  
in the string  $A L A B A S T E R$  ?
- 10) What is the sum of the distinct roots/zeros of the equation 10) \_\_\_\_\_  
 $|x|^2 + |x| - 6 = 0$  where  $|x|$  denotes the absolute value of  $x$  ?

(OVER)

- 11) If the point  $(x, -4)$  lies on the straight line joining the points  $(-4, 0)$  and  $(0, 8)$  in the  $xy$ -plane, what is the value of  $x$ ? 11) \_\_\_\_\_
- 12) If  $a = \log_8(225)$  and  $b = \log_2(15)$ , then 12) \_\_\_\_\_  
 (a)  $a = \frac{b}{2}$       (b)  $a = \frac{2b}{3}$       (c)  $a = b$   
 (d)  $a = 2b$       (e)  $a = \frac{3b}{2}$
- 13) If four times the reciprocal of the circumference of a circle equals the diameter of the circle, what is the circle's area? 13) \_\_\_\_\_
- 14) If the factorization of  $x^2 + 5x + m$  is  $(x+n)^2$ , then  $\frac{m}{n}$  is 14) \_\_\_\_\_  
 (a)  $-\frac{5}{2}$       (b)  $\frac{2}{5}$       (c)  $-\frac{2}{5}$       (d)  $\frac{5}{2}$
- 15) Express  $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h}$  as a function of  $x$  only. 15) \_\_\_\_\_
- 16) Solve the following system of equations for  $x$  and  $y$ : 16) \_\_\_\_\_
- $$\begin{cases} 2x - y = 5 \\ \frac{2y}{x} - \frac{x}{y} = 1 \end{cases}$$
- 17) If  $|x + 3 + 4i| = 2$ , then the maximum value of  $|x|$  is 17) \_\_\_\_\_  
 (a) 3      (b) 5      (c) 7      (d) 9
- 18) A right circular cylinder has volume  $54\pi \text{ in}^3$ . If the radius of the cylinder is increased by 2 inches but the height remains unchanged, its volume would be  $150\pi \text{ in}^3$ . What is the height of the cylinder? 18) \_\_\_\_\_
- 19) If  $abc \neq 0$  and  $a+b+c = 0$ , then express  $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$  as an integer. 19) \_\_\_\_\_
- 20) Assume the diagonal of a cube has length  $a$ . Express the surface area of the cube as a function of  $a$ . 20) \_\_\_\_\_